Magnetic Field of an Accretion Disk and the Formation of Relativistic Jets

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Magnetic Fields and the Formation of Relativistic Jets

Abstract: When matter falls towards a black hole, it will form a disk around the black hole known as the accretion disk. Accretion disks give rise to relativistic jets coming from the central object. The hypothesis is that the twisting of the magnetic field produced by the accretion disk collimates the outflow along the rotation axis of the central object, so that when conditions are suitable, a jet will emerge from each face of the accretion disk. Along with the theory that the magnetic field is twisted due to the spin of the black hole, a particle’s motion in the z direction, that of the rotation axis, can be associated with the formation of relativistic jets. We have constructed a model of the magnetic field produced by an accretion disk around a black hole in an active galactic nucleus, and used it to show that particles coming off the accretion disk move in the z direction. A computer program was created to simulate the motion of a particle in the magnetic field. The particle is shown to leave its circular motion and move in the z direction.

Magnetic Field for an Accretion Disk

The magnetic field due to a current carrying loop is described by the Biot-Savart Law. The Biot-Savart Law (1.11) gives an expression for the magnetic field contribution, $d\mathbf{B}$, from the current source, $I d\mathbf{s}$, with $\mu_0$ (1.12) as the permeability of free space constant. Let $r$ denote the distance from the current source to the field point, and $\mathbf{r}$ as the corresponding unit vector.

$$d\mathbf{B} = \frac{\mu_0 I d\mathbf{s} \times \hat{\mathbf{r}}}{4\pi r^2}$$  \hspace{1cm} (1.11)

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$  \hspace{1cm} (1.12)
A circular loop of radius R in the $xy$-plane carries a current $I$, as shown in Figure 1.13. The loop is symmetrical about the $x$ and $y$ axes.

![Figure 1.13](image)

We have imagined that the source point not be on the $z$-axis, but somewhere in the $yz$-plane. Thus the field point’s position vector is given by $\vec{r}_p = y\hat{j} + z\hat{k}$ and the position of the field point from the loop is given by $\vec{r} = -R\cos(\phi')\hat{i} + (y - R\sin(\phi'))\hat{j} + z\hat{k}$.

In Cartesian coordinates, the differential current element can be written as

$$ld\hat{s} = lRd\phi'(-\sin(\phi')\hat{i} + \cos(\phi')\hat{j}),$$

and the cross product as

$$ld\hat{s} \times \hat{r} = lR\left(z\cos(\phi')\hat{i} + z\sin(\phi')\hat{j} + (R - y\sin(\phi'))\hat{k}\right)d\phi'.$$

The $x$, $y$, and $z$ components of $\vec{B}$ at position $\vec{r}_p$ are given by

$$B_x = \frac{\mu_0 lR}{4\pi} \int_0^{2\pi} \frac{z\cos(\phi') d\phi'}{((-R\cos(\phi'))^2 + (y - R\sin(\phi'))^2 + z^2)^{3/2}}.$$
The magnetic field in the \( x \) direction is determined to be zero as we have chosen field points in the \( yz \)-plane and the loop is symmetrical about the \( x \) and \( y \) axes. We then computed the magnitude of the magnetic field in the \( yz \)-plane in terms of \( y \) and \( z \) coordinates. Using our FORTRAN computer program, we computed these integrals given by Biot-Savart by summing the magnetic field over small portions of the loop. Figure 1.14 shows the lines of magnetic force for a single loop, interpolated from the magnetic field data given by our program. The values for the variables were chosen for graphical convenience.

\[
\mathbf{B}_y = \frac{\mu_0 I R}{4\pi} \int_0^{2\pi} \frac{z \sin(\varphi') \, d\varphi'}{((-R \cos(\varphi'))^2 + (y - R \sin(\varphi'))^2 + z^2)^\frac{3}{2}}
\]

\[
\mathbf{B}_z = \frac{\mu_0 I R}{4\pi} \int_0^{2\pi} \frac{R - y \sin(\varphi') \, d\varphi'}{((-R \cos(\varphi'))^2 + (y - R \sin(\varphi'))^2 + z^2)^\frac{3}{2}}
\]

To obtain magnetic field data for a disk in the \( xy \)-plane with surface current, a simple addition to our original program was made. We now summed our original integration for many rings in between the inner and outer radius of the disk. Once again for graphical purposes, we have assumed the disk to have a constant current of 1.0 A, an inner radius of 1.0, and an outer radius

**Figure 1.14** Lines of magnetic force in the \( yz \)-plane, for a current carrying loop of radius 1.0 and current 1.0 A.
of 2.0. Figure 1.15 illustrates the lines of magnetic force for this disk. We noticed that the magnetic field gets much larger near the disk and at positions above the disk the magnetic field is primarily in the $y$ direction. For an accretion disk in active galactic nuclei, the current varies as $\frac{1}{\sqrt{R}}$. The magnetic field lines due to an accretion disk are shown in Figure 1.16. The spacing of the lines is inversely proportional to their magnitude of magnetic field.

![Figure 1.15 Lines of magnetic force for a disk in the $yz$-plane with inner radius 1.0 and outer radius 2.0 and constant current.](image1)

![Figure 1.16 Lines of magnetic force for a disk in the $yz$-plane with inner radius 1.0 and outer radius 2.0 and current varying as $\frac{1}{\sqrt{R}}$.](image2)

The magnetic fields are plotted in 2D, but can easily be thought of as a representation for the 3D image as the loop or disk has rotational symmetry about the $z$-axis.

**Formation of Relativistic Jets**

Some accretion disks produce twin jets of highly collimated and fast outflows that emerge from opposite sides of the disk. These jets are called relativistic jets. The direction of the jet ejection
is determined by the spin axis of the black hole. Around the black hole, there is a mathematically defined surface called the event horizon. The event horizon is a boundary in spacetime where events inside cannot affect an outside observer. It is the boundary at which the gravitational pull becomes so great that no matter or radiation can escape. The radius of the black hole is defined by the event horizon. For a non-rotating black hole, the radius is proportional to the mass of the black hole. A rotating, uncharged, spherically-symmetric black hole is known as a Kerr metric. The Kerr-Newman metric is a charged, rotating black hole. We assumed a Kerr metric.

The current model states that relativistic jets are likely formed by the twisting of the magnetic field lines from the accretion disk causing matter to collimate along the rotation axis of the central object. Magnetic field lines are likely twisted by the spin of the black hole twisting space.\(^1\) If a particle is able to move in the vertical direction and we include the twisting of the magnetic field, this may be an accurate model for how relativistic jets form. In our model, we showed that a particle near the accretion disk is able to get out of its circular motion and take off in a vertical trajectory.

**Motion of Particles**

A particle’s motion in the magnetic field due to an accretion disk is simulated to see that particles are redirected from their circular orbit into a vertical trajectory. The force on the particle due to electromagnetic fields is the Lorentz force 1.17. The Lorentz force equation was used along with Newton’s second law 1.18 to describe the velocity of the particle at a certain point in time.

\[
F = q(\vec{v} \times \vec{B}) \tag{1.17}
\]

\[
F = \frac{d\vec{p}}{dt} \tag{1.18}
\]
For a sufficiently small $\Delta t$, the change in velocity remains constant over that period of time. The motion of the particle is thus tracked using the change in velocity found by the equations:

\[
\begin{align*}
\left(\frac{\Delta v_x}{\Delta t}\right) &= \frac{q}{m} \left(\begin{array}{c} v_x \\ v_y \\ v_z \end{array}\right) \times \left(\begin{array}{c} B_x \\ B_y \\ B_z \end{array}\right) = \frac{q}{m} \left(\begin{array}{c} v_y B_z - v_z B_y \\ v_z B_x - v_x B_z \\ v_x B_y - v_y B_x \end{array}\right) \\
\left(\frac{\Delta v_y}{\Delta t}\right) &= \frac{q}{m} \left(\begin{array}{c} v_x \\ v_y \\ v_z \end{array}\right) \times \left(\begin{array}{c} B_x \\ B_y \\ B_z \end{array}\right) = \frac{q}{m} \left(\begin{array}{c} v_y B_z - v_z B_y \\ v_z B_x - v_x B_z \\ v_x B_y - v_y B_x \end{array}\right) \\
\left(\frac{\Delta v_z}{\Delta t}\right) &= \frac{q}{m} \left(\begin{array}{c} v_x \\ v_y \\ v_z \end{array}\right) \times \left(\begin{array}{c} B_x \\ B_y \\ B_z \end{array}\right) = \frac{q}{m} \left(\begin{array}{c} v_y B_z - v_z B_y \\ v_z B_x - v_x B_z \\ v_x B_y - v_y B_x \end{array}\right)
\end{align*}
\]

Constants and initial conditions are defined so that the motion of a particle can be observed over one hundred steps of $\Delta t$.

\[
\Delta t = 5 \times 10^{-10} \text{ s}
\]

\[
\frac{q}{m} = 1.76 \times 10^{11} \text{ C.kg}^{-1}
\]

\[
x_0 = 0.0
\]

\[
y_0 = 1.1 \rightarrow 1.5
\]

\[
z_0 = 0.2
\]

\[
\vec{v} = 10^8 \hat{i} \text{ m/s}
\]

\[
I = \frac{20.0}{\sqrt{R}} \text{ A}
\]
First the initial conditions and position are read into our computer program. For this position, the magnetic field in each direction is computed. Then using equations (1.19), the change in velocity is found. The particle’s velocity is now given by $\vec{v}_f = \vec{v}_i + \Delta \vec{v}$. The particle’s new position $\vec{R}_f$ is thus given by $\vec{R}_f = \vec{R}_i + \vec{v} \times \Delta t$. This process is repeated for one hundred steps of $\Delta t$.

Random steps were checked by hand calculations in which the magnetic field and change in position were both verified.

The position vector was recorded at each step and plotted in Mathematica to obtain a visual representation of the particles motion in this magnetic field. Particles with initial positions from $y_0 = 1.1 \rightarrow 1.5$ are plotted.

![Figure 1.20](image-url) **Figure 1.20** Three-dimensional image of a particles motion in the magnetic field produced by an accretion disk. The inner and outer radius of the accretion disk is shown by the blue rings. The axis of rotation is given by the red line.
We can see that particles that start near the accretion disk quickly leave their circular motion and move in the z direction.

**Conclusion**

The formation of relativistic jets has two key components. The first component being that the particles are able to move upwards along the axis of rotation without continuing in a circular motion. The second being the twisting of the magnetic field lines due to the spin of the black hole. We have successfully constructed a model that shows a particle’s ability to move vertically in the same direction as the relativistic jets.
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Works Cited


