Notre Dame Physics Department Preliminary Qualifying Examination

SAMPLE **(not for distribution)**

Part II

Each problem will be graded on a scale of 0-4 points. You are asked to do any 8 of the problems.

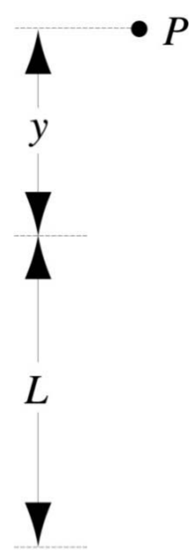
Clearly indicate your choices, by listing here the two problems that you are not going to attempt:
a) ________ b) _______.

Use only **PEN** for this test. Show all your work on separate pages for each problem. Please use only one side of the paper to work the problems!
Collect your work together in numerical order (number each page) by problem when you finish, including your equation sheet at the end, use the envelope provided to store your work and the exam. Good luck!

Please confirm your student ID number: **Master.**

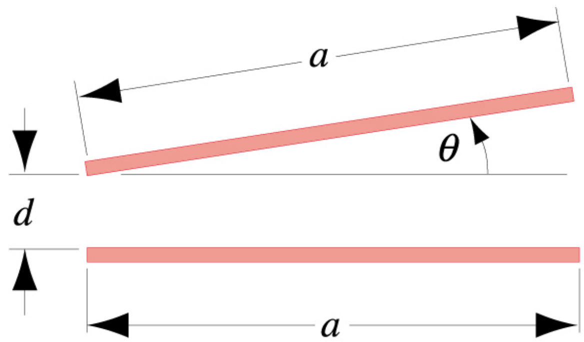
DO NOT WRITE YOUR NAME!

1. A charge per unit length λ is distributed uniformly along a thin rod of length L. (*a*) Determine the potential (chosen to be zero at infinity) at a point P a distance y from one end of the rod and in line with it (see the figure). (*b*) Use the result of (*a*) to compute the component of the electric field at P in the y direction (along the rod). (*c*) Determine the component of the electric field at P in a direction perpendicular to the rod.

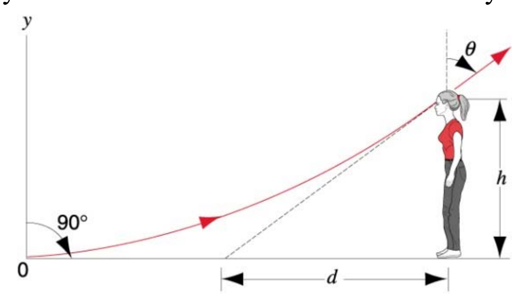


2. A stack of polarizing sheets is arranged so that the angle between any two adjacent sheets is α. The sheets are arranged so that N sheets rotate the plane of polarization by θ, where $\theta = N\alpha$. Calculate the fraction of light that will pass through the stack in the limit as $N\to\infty$. Assume that θ is fixed, so $\alpha \to 0$.

3. A capacitor has square plates, each of side a, making an angle θ with each other as shown in the figure. (a) Derive an expression for the capacitance for small values of θ. (b) Assume that the capacitor is charged to a constant charge q at $\theta = 0°$. Derive an expression for the work required to tilt the upper plate to a small angle θ. (c) Assume the capacitor is connected to a constant voltage V at $\theta = 0°$. Derive an expression for the work required to tilt the upper plate to a small angle θ.



4. You stand at one end of a long runway. A vertical temperature gradient in the air has resulted in the index of refraction of the air above the runway to vary with height **y** according to $n = n_0(1 + ay)$, where n_0 is the index of refraction at the runway surface and $a = 1.5 \text{ X } 10^{-6}\ \text{m}^{-1}$. Your eyes are at a height $h = 1.7$ m above the runway. Beyond what horizontal distance d can you not see the runway?



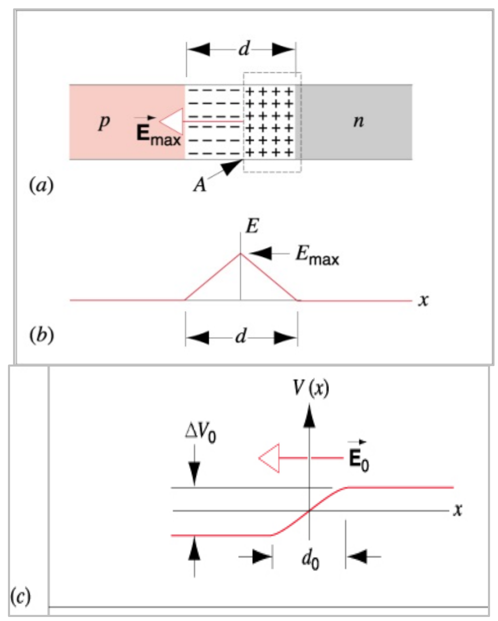
5. A particle is confined between rigid walls located at $x = 0$ and $x = L$. For the $n = 4$ energy state, (a) sketch the probability density curve for the particle's location. Calculate the approximate probabilities of finding the particle within a region $\Delta x = 0.0003L$ when (b) Δx is located at $x = L\vert 8$ and (c) at $x = 3L/16$. Refer to your figure to see whether or not your results seem reasonable. (Hint: No integration is necessary.)

6. Two long, parallel wires, each of radius a, whose centers are a distance d apart carry equal currents in opposite directions. Neglecting the flux within the wires themselves, derive an expression for the inductance of a length l of such a pair of wires.

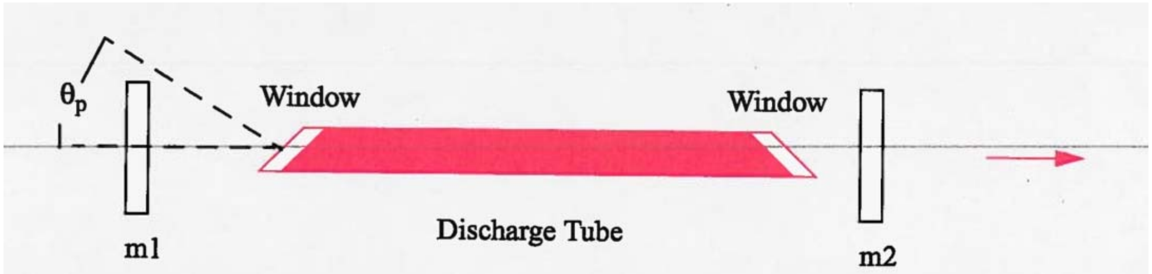
7. The recessional speeds of galaxies and quasars at great distances are close to the speed of light, so that the relativistic Doppler shift formula must be used. The redshift is reported as z, where $z = \Delta\lambda/\lambda_0$ is the (fractional) red shift. (*a*) Derive an expression for the recessional speed parameter $\beta = \mathbf{v}/c$ in terms of **z.** (*b*) One of the most distant quasars detected has $z = 4.43$. Calculate its speed parameter. (*c*) Find the distance to the quasar, assuming that Hubble's law is valid to these distances and that H = 72 km/(s Mpc).
Hint: You will need to derive an expression for the relativistic Doppler shift using the Lorentz contraction ($L = L_0/\gamma$) and time dilation ($\Delta t = \gamma\Delta t_0$) formulae. An alternative (and easier) derivation follows from the fact that the photon momentum and energy transform as a 4-vector.

8. The time-independent portion of a wave function for an electron of energy $E = h\omega/4\pi$ is $\varphi(x) = A_0 \exp\left(-\pi m\omega x^2/h\right)$, where A_0 is a constant. Substitute this into Schrödinger equation and find the equation for the potential energy $U(x)$. What does this potential energy represent?

9. A silicon-based *pn* junction has an equal concentration n_0 of donor and acceptor atoms. Its depletion zone, of width d, is symmetrical about the junction plane, as Figs. (*a*) and (*b*) show. (*a*) Derive an expression for $E_{\max}$, the maximum intensity of the electric field in the depletion zone.
(*b*) Derive an expression for ΔV_0, the potential difference that exists across the depletion zone; see Fig. (*c*).
(*c*) Assume that $n_0 = 3 \times 10^{22}\,\mathrm{m}^{-3}$ and that ΔV_0 is measured to be 0.6 V. Calculate the width of the depletion zone.
(*d*) Using this value of d, calculate the value of $E_{\max}$. The dielectric constant of silicon is K_e (=12).
The donor and acceptor atoms move or remove charge from the depletion zone, such that there is a constant charge density that changes sign in the middle. You can assume that the device extends to infinity in vertical direction.



10. The figure schematically illustrates a He-Ne gas laser. Various statements describing the several features of the elements of the physics involved are enclosed in braces {}, typed in blue, and separated by commas. Select the correct statement in each case and write it into your paper. **(Note that this problem will be graded by computing the number of correct responses, minus the number of incorrect responses.)**



The ends of the discharge tube are sealed with windows which are mounted at a special angle θ_p specified by Brewster's law. At this angle the reflected wave is {*linearly polarized, circularly polarized*}, {*extinguished, transmitted*}. The refracted wave is {*transmitted with very small attenuation, extinguished*}, {*partially linearly polarized, partially circularly polarized*}.

The mirror m1 is {*substantially transmissive, nearly perfectly reflecting*} and is curved to focus the light {*inside (real image), outside (virtual image)*} the discharge tube. As a result, this mirror is {*perfectly flat, slightly convex, slightly concave*}.

The mirror m2 is {*substantially transmissive, partially transmissive*} and is curved to focus the light {*inside (real image), outside (virtual image)*} the discharge tube. As a result, this mirror is {*perfectly flat, slightly convex, slightly concave*}.

The laser light is polarized {*in the plane of the figure, perpendicular to the plane of the figure*}. Mounting the windows perpendicular to the beam direction would result in {*higher, very much lower, non-existent*}output power.

In this laser, the gas discharge results in the excitation of He atoms to a level which is {*metastable, rapidly decaying to the ground state*}. The Ne atoms have an energy level which is {*near, distant from*} the excited He level, and the energy is transmitted to the {*He, Ne*} atoms by collisions. The {*He, Ne*} atoms experience stimulated emission which results in the {*increase, decrease*} of the number of photons which are {*phase coherent, randomly phased*} relative to the stimulating photon. Photons which are emitted perpendicular to the beam direction are {*reflected into the beam, lost*}. Typically, photons are reflected from the mirrors {*only once, many times*} before leaving the laser.