

# Classical Mechanics

## Linear motion

For constant acceleration  $\mathbf{a}$ , if at  $t = 0 \mathbf{r} = \mathbf{r}_0$  and  $\mathbf{v} = \mathbf{v}_0$ :  $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$   
 $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a}t^2$

Circular motion at constant speed  $a = v^2/r = \omega^2 r$  (Centripetal acceleration, points towards center of circle,  $\omega$  is angular speed in radians per second)

Adding relative velocities ("wrt" is short for "with respect to"):  $\mathbf{v}_{A \text{ wrt } B} + \mathbf{v}_{B \text{ wrt } C} = \mathbf{v}_{A \text{ wrt } C}$

$\sum \mathbf{F} = 0 \Leftrightarrow \mathbf{a} = 0$  (Newton's first law)

$\mathbf{F} = m\mathbf{a}$  or  $\mathbf{F} = d\mathbf{p}/dt$  (Newton's second law)  $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$  (Newton's third law)

$\mathbf{p} = m\mathbf{v}$  (momentum)

$$\mathbf{r}_{\text{cm}} = \frac{\sum m_i \mathbf{r}_i}{\sum m_i} \quad (\text{position of center of mass})$$

$$\mathbf{F} = -k\mathbf{x} \quad (\text{spring force}) \quad f \leq \mu N \quad (\text{Friction force relative to Normal force})$$

$$\mathbf{F} = -\frac{GMm}{r^2} \hat{\mathbf{r}} \quad (\text{gravitational force between two particles})$$

$$W = \int \mathbf{F} \cdot d\mathbf{r} \quad (\text{work done by force } \mathbf{F})$$

$$W_{\text{other}} = \Delta E = E_F - E_I \quad E = KE + PE \quad (\text{work-energy theorem})$$

$$\vec{F} = -\nabla U \quad (\text{force derived from potential energy})$$

$$\text{Potential Energies: } U = \frac{1}{2} k(x - x_0)^2 \quad (\text{spring force})$$

$$U = \frac{-GM(r)m}{r} \quad (\text{gravitational, general}) \quad U = mgh \quad (\text{gravitational, near Earth})$$

$$\text{Rotational motion} \quad \omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \quad (\text{constant angular acceleration } \alpha)$$

$$v = \omega R \quad (\text{rolling without slipping})$$

$$\tau = \mathbf{r} \times \mathbf{F} = \frac{d\mathbf{L}}{dt} = I\boldsymbol{\alpha} \quad |\tau| = rF \sin(\phi) = Fr_{\perp} \quad (\text{torque equations})$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \quad |\mathbf{L}| = mvrsin(\phi) \quad (\text{angular momentum of point particle})$$

$$\mathbf{L} = I\boldsymbol{\omega} \quad (\text{angular momentum for solid object})$$

$$I_{\text{I}} = I_{\text{c.m.}} + Md^2 \quad (\text{parallel axis theorem})$$

$$I = \frac{1}{2}MR^2 \quad (\text{cylinder around center}) \quad I = \frac{2}{5}MR^2 \quad (\text{solid sphere around center})$$

$$I = \frac{1}{12}ML^2 \quad (\text{rod around center}) \quad I = \frac{1}{3}ML^2 \quad (\text{rod around end})$$

$$KE = \frac{1}{2}M_{\text{tot}}v_{CM}^2 + \frac{1}{2}I_{CM}\omega^2 \quad (\text{kinetic energy for object moving and rolling})$$

$$KE = \frac{1}{2}I_{\text{pivot}}\omega^2 \quad (\text{kinetic energy for object rotating around a fixed pivot})$$

$$I_{ab} \equiv \sum_k m_k (\delta_{ab} \mathbf{r}_k^2 - r_k^a r_k^b) \quad (\text{general moment of inertia})$$

## Simple Harmonic Motion

$$\omega = \sqrt{k/m} \quad x = A \cos(\omega t + \phi) \quad \omega = 2\pi f$$

$$v = -A\omega \sin(\omega t + \phi) \quad T = \frac{2\pi}{\omega}$$

## Coupled oscillators

$$\mathbf{M} \ddot{\mathbf{q}} = -\mathbf{K} \mathbf{q}$$

$$(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{a} = 0. \quad (\text{normal modes})$$

$$\text{Orbit in Newtonian potential } U = -C/r, \quad C > 0, \quad E < 0$$

$$p = \frac{L^2}{mC}, \quad e = \sqrt{1 + \frac{2EL^2}{mC^2}}, \quad \text{Period} = \frac{2\pi a^{3/2}}{\sqrt{C/m}}$$

$$V_{\text{eff}} = kr^s + \frac{l^2}{2mr^2}$$

$$l = mR^2\dot{\theta}$$

$$T^2 \propto a^3$$

$$\omega = 2\pi/T$$

$$\mathbf{v} \equiv \dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\dot{\phi}\hat{\phi}$$

$$e = \sqrt{1 + \frac{2El^2}{mk^2}} \quad \mu = \frac{m_1 m_2}{m_1 + m_2} \quad (\text{reduced mass})$$

## Rotating frames

$$m\ddot{\mathbf{r}} = \mathbf{F} + \underbrace{2m\dot{\mathbf{r}} \times \boldsymbol{\Omega}}_{\text{(coriolis force)}} + \underbrace{m(\boldsymbol{\Omega} \times \mathbf{r}) \times \boldsymbol{\Omega}}_{\text{(centrifugal force)}}$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

## Lagrangian and Hamiltonian mechanics

$$\mathcal{L} = T - V \quad (\text{T} = \text{KE})$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \frac{\partial \mathcal{L}}{\partial \theta}$$

$$H = T + V$$

$$H = -\mathcal{L} + p_i \dot{q}_i$$

$$p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$$

$$-\dot{p}_i = \frac{\partial H}{\partial q_i}; \dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$\frac{dH}{dt} = -\frac{\partial \mathcal{L}}{\partial t}$$

$$T = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2 + r^2 \sin^2(\theta)\dot{\phi}^2)$$

$$H = \frac{1}{2m}(p_r^2 + \frac{1}{r^2}p_\theta^2 + \frac{1}{r^2 \sin^2(\theta)}p_\phi^2) \quad (\text{spherical, cylindrical coords})$$

$$H = \frac{1}{2m}(p_r^2 + \frac{1}{r^2}p_\theta^2 + p_z^2)$$

## Statistical Mechanics

First law:  $dU = dQ + dW_{on}$

Second law:  $dS \geq 0$

Entropy:

- $S \equiv k \ln \Omega$
- $S_{\text{total}} = S_1 + S_2$

Temperature, pressure, and chemical potential:

- $T^{-1} \equiv \left(\frac{dS}{dU}\right)_{V,N}$
- $p \equiv T \left(\frac{dS}{dV}\right)_{U,N}$
- $-\mu \equiv T \left(\frac{dS}{dN}\right)_{U,V}$   
 $= -\left(\frac{dF}{dN}\right)_{T,V}$

Fundamental relation:

$$dS = \frac{1}{T}dU + \frac{p}{T}dV - \frac{\mu}{T}dN$$

Ideal Gas Law:

$$pV = NkT$$

Adiabat

$$PV^\gamma = \text{constant}$$

### Thermodynamic Potentials:

- $F(T, V) = U - TS$  Free Energy
- $G(T, p) = U - TS + pV$  Gibbs
- $H(S, p) = U + pV$  Enthalpy

### Work:

$$dW_{on} = -dW_{by} = -pdV$$

### First Law:

$$dU = dQ - pdV$$

### Equipartition:

$$U = \left(\frac{1}{2}\right) k_B T \text{ per quadratic degree of freedom}$$

### Heat Capacity:

- $C \equiv \frac{dQ}{dT}$
- Constant volume:  $C_V = \frac{dU}{dT}$
- Constant pressure:  
 $C_p = \frac{dU}{dT} + p \frac{dV}{dT}$

- Alternative forms

$$C_{P,V} = T \left( \frac{\partial S}{\partial T} \right)_{P,V}$$

### Thermodynamic Processes:

- Isothermal:  $T = \text{const.}$
- Isobaric:  $p = \text{const.}$
- Isochoric:  $V = \text{const.}$
- Adiabatic:  $Q = 0$

### Boltzmann Factor:

- $P(E_i) = Z^{-1} e^{-\frac{E_i}{k_B T}}$
- $Z = \sum_i e^{-E_i/k_B T}$

### Thermal Radiation

- $J = \sigma_B T^4$

### Counting particles

- Distinguishable:  $\Omega = M^N$
- Indistinguishable:  $\Omega = \frac{M^N}{N!}$
- $q$  quanta in  $N$  oscillators:  
 $\binom{N-1+q}{q} = \frac{(N-1+q)!}{q!(N-1)!}$

### Compressability

$$\bullet \quad \kappa_{T,S} = -\frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_{T,S}$$

### Coefficient of thermal expansion

$$\bullet \quad \beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_P$$