Electrostatics:

$\vec{F} = q\vec{E}$, where

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{(\vec{r}-\vec{r}')\,q_i}{|\vec{r}-\vec{r}'|^3} = \frac{1}{4\pi\epsilon_0}\int \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,\rho(\vec{r}')\,\mathrm{d}^3x'$$

ϵ_0 =permittivity of free space $= 8.854\times10^{-12}$ $\mathrm{C^2/(N{\cdot}m^2)}$

$$\frac{1}{4\pi\epsilon_0} = 8.988\times10^9\ \mathrm{N{\cdot}m^2/C^2}$$

$$V(\vec{r}) = V(\vec{r}_0) - \int_{\vec{r}_0}^{\vec{r}} \vec{E}(\vec{r}')\cdot \mathrm{d}\vec{\ell}' = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\mathrm{d}^3x'$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}, \qquad \vec{\nabla}\times\vec{E} = 0, \qquad \vec{E} = -\vec{\nabla}V$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}\ \text{(Poisson's Eq.)}, \qquad \rho = 0 \implies \nabla^2 V = 0\ \text{(Laplace's Eq.)}$$

$$W = \frac{1}{2}\int \mathrm{d}^3x\rho(\vec{r})V(\vec{r}) = \frac{1}{2}\epsilon_0\int|\vec{E}|^2\,\mathrm{d}^3x$$

$\overrightarrow{E} = 0$ in conductors

Conductors:

Just outside, $\vec{E} = \dfrac{\sigma}{\epsilon_0}\hat{n}$

Pressure on surface: $\frac{1}{2}\sigma|\vec{E}|_{\text{outside}}$

Two-conductor system with charges Q and $-Q$: $Q = CV$, $W = \frac{1}{2}CV^2$

Electric Fields in Matter:

Electric Dipoles:

$$\vec{p} = \int d^3x\,\rho(\vec{r})\,\vec{r} \qquad V_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2} \qquad \mathbf{E}_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}].$$

$\rho_{\text{dip}}(\vec{r}) = -\vec{p}\cdot\vec{\nabla}_{\vec{r}}\,\delta^3(\vec{r}-\vec{r}_d)$, where $\vec{r}_d$ = position of dipole

$\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E} = \vec{\nabla}(\vec{p}\cdot\vec{E})$ (force on a dipole)

$\vec{\tau} = \vec{p}\times\vec{E}$ (torque on a dipole)

$U = -\vec{p}\cdot\vec{E}$

Electrically Polarizable Materials:

$\vec{P}(\vec{r})$ = polarization = electric dipole moment per unit volume

$\rho_{\text{bound}} = -\nabla\cdot\vec{P}$, $\quad\sigma_{\text{bound}} = \vec{P}\cdot\hat{n}$

$\vec{D} \equiv \epsilon_0\vec{E} + \vec{P}$, $\quad\vec{\nabla}\cdot\vec{D} = \rho_{\text{free}}$, $\quad\vec{\nabla}\times\vec{E} = 0$ (for statics)

Boundary conditions:

$$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{\sigma}{\epsilon_0} \qquad D^{\perp}_{\text{above}} - D^{\perp}_{\text{below}} = \sigma_{\text{free}}$$

$$\vec{E}^{\parallel}_{\text{above}} - \vec{E}^{\parallel}_{\text{below}} = 0 \qquad \vec{D}^{\parallel}_{\text{above}} - \vec{D}^{\parallel}_{\text{below}} = \vec{P}^{\parallel}_{\text{above}} - \vec{P}^{\parallel}_{\text{below}}$$

Capacitance

Parallel-plate: $C = \varepsilon_0\dfrac{A}{d}$

Spherical: $C = 4\pi\varepsilon_0\dfrac{ab}{b-a}$

Cylindrical: $C = 2\pi\varepsilon_0\dfrac{L}{\ln(b/a)}$ (for a length L)

Magnetostatics:

Magnetic Force:

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

$$\vec{F} = \int I\mathrm{d}\vec{\ell}\times\vec{B}$$

Biot-Savart Law:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}I\int\frac{\mathrm{d}\vec{\ell}'\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}')\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,\mathrm{d}^3x$$

Infinitely long straight wire: $\vec{B} = \dfrac{\mu_0 I}{2\pi r}\,\hat{\phi}$

Infinitely long tightly wound solenoid: $\vec{B} = \mu_0 n I_0\,\hat{z}$, where n = turns per unit length

Loop of current on axis: $\vec{B}(0,0,z) = \dfrac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}\,\hat{z}$

Infinite current sheet: $\vec{B}(\vec{r}) = \dfrac{1}{2}\mu_0\vec{K}\times\hat{n}$, $\hat{n}$ = unit normal toward $\vec{r}$

Vector Potential:

$$\vec{A}(\vec{r})_{\text{coul}} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, \mathrm{d}^3x' \ , \qquad \vec{B} = \vec{\nabla} \times \vec{A} \ , \qquad \vec{\nabla} \cdot \vec{A}_{\text{coul}} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

Ampère's Law:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \ , \ \text{ or equivalently } \ \int_P \vec{B} \cdot \mathrm{d}\vec{\ell} = \mu_0 I_{\text{enc}}$$

Magnetic dipole

$$\vec{m} = \frac{1}{2} I \int_P \vec{r} \times \mathrm{d}\vec{\ell} = \frac{1}{2} \int \mathrm{d}^3x \, \vec{r} \times \vec{J} = I\vec{a} \ ,$$

$$\vec{B}_{\text{dip}}(\vec{r}) = \frac{\mu_0}{4\pi} \vec{\nabla} \times \frac{\vec{m} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m}}{r^3} + \frac{2\mu_0}{3} \vec{m} \, \delta^3(\vec{r})$$

$$\vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B})$$
$$\vec{\tau} = \vec{m} \times \vec{B}$$
$$U = -\vec{m} \cdot \vec{B}$$

Maxwell's Equations:

$$\text{(i)} \quad \vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho \qquad \text{(iii)} \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \ ,$$

$$\text{(ii)} \quad \vec{\nabla} \cdot \vec{B} = 0 \qquad \text{(iv)} \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

where $\mu_0 \epsilon_0 = \dfrac{1}{c^2}$

Lorentz force law: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Charge conservation: $\dfrac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{J}$

Poynting vector (flow of energy): $\vec{S} = \dfrac{1}{\mu_0} \vec{E} \times \vec{B}$

Electromagnetic energy & conservation

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{J}.$$

$$u = \frac{1}{2} \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right)$$

Magnetically Polarizable Materials:

$\vec{M}(\vec{r})$ = magnetization = magnetic dipole moment per unit volume

$$\vec{J}_{\text{bound}} = \vec{\nabla} \times \vec{M} \ , \qquad \vec{K}_{\text{bound}} = \vec{M} \times \hat{n}$$

$$\vec{H} \equiv \frac{1}{\mu_0} \vec{B} - \vec{M} \ , \qquad \vec{\nabla} \times \vec{H} = \vec{J}_{\text{free}} \ , \qquad \vec{\nabla} \cdot \vec{B} = 0$$

Boundary conditions:

$$B^{\perp}_{\text{above}} - B^{\perp}_{\text{below}} = 0 \qquad H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}})$$

$$\vec{B}^{\parallel}_{\text{above}} - \vec{B}^{\parallel}_{\text{below}} = \mu_0(\vec{K} \times \hat{n}) \qquad \vec{H}^{\parallel}_{\text{above}} - \vec{H}^{\parallel}_{\text{below}} = \vec{K}_{\text{free}} \times \hat{n}$$

Current, Resistance, and Ohm's Law:

$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$, where σ = conductivity. $\rho = 1/\sigma$ = resistivity

Resistors: $V = IR$, $\qquad P = IV = I^2R = V^2/R$

Resistance in a wire: $R = \dfrac{\ell}{A} \rho$, where ℓ = length, A = cross-sectional area, and ρ = resistivity

Charging an RC circuit: $I = \dfrac{V_0}{R} e^{-t/RC}$, $\qquad Q = CV_0 \left[1 - e^{-t/RC} \right]$

Inductance:

Universal flux rule: Whenever the flux through a loop changes, whether due to a changing $\vec{B}$ or motion of the loop, $\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$, where Φ_B is the magnetic flux through the loop

integral form $\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$

Mutual inductance: $\Phi_2 = M_{21} I_1$, M_{21} = mutual inductance

(Franz) Neumann's formula: $M_{21} = M_{12} = \frac{\mu_0}{4\pi} \oint_{P_1} \oint_{P_2} \frac{\mathrm{d}\vec{\ell}_1 \cdot \mathrm{d}\vec{\ell}_2}{|\vec{r}_1 - \vec{r}_2|}$

Self inductance: $\Phi = LI$, $\quad \mathcal{E} = -L\frac{\mathrm{d}I}{\mathrm{d}t}$; $\quad L$ = inductance

Self inductance of a solenoid: $L = n^2 \mu_0 \mathcal{V}$, where n = number of turns per length, $\mathcal{V}$ = volume

Rising current in an RL circuit: $I = \frac{V_0}{R}\left[1 - e^{\frac{R}{L}t}\right]$

Variable separation in spherical coords

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$$

Image charges for grounded sphere:

$$q' = -\frac{R}{a}q, \qquad b = \frac{R^2}{a}$$

a = distance to sphere,
q = initial charge,
R = radius
b = image charge location

Electromagnetic Waves:

Wave Equations: $\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$, $\quad \nabla^2 \vec{B} - \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} = 0$

Linearly Polarized Plane Waves:

$\vec{E}(\vec{r},t) = \tilde{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}\,\hat{n}$, where $\tilde{E}_0$ is a complex amplitude, $\hat{n}$ is a unit vector, and $\omega/|\vec{k}| = v_{\text{phase}} = c$.

$\hat{n} \cdot \vec{k} = 0$ (transverse wave)

$\vec{B} = \frac{1}{c}\hat{\boldsymbol{k}} \times \vec{E}$

$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} = uc\,\hat{z}$, $\qquad I$ (intensity) $= \left\langle |\vec{S}| \right\rangle = \frac{1}{2}\epsilon_0 E_0^2$

momentum density $\quad \mathbf{g} = \frac{1}{c^2}\mathbf{S}.$

alternate form for waves

$$\mathbf{E}(\mathbf{r},t) = E_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)\,\hat{\mathbf{n}},$$

$$\mathbf{B}(\mathbf{r},t) = \frac{1}{c}E_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)(\hat{\mathbf{k}} \times \hat{\mathbf{n}}).$$