Notre Dame Physics Department Preliminary Qualifying Examination
SAMPLE
Part II

Each problem will be graded on a scale of 0-4 points. You are asked to do any 8 of the problems.

Clearly indicate your choices, by listing here the two problems that you are not going to attempt: a) _______ b) _______.

Use only PEN for this test. Show all your work on separate pages for each problem. Please use only one side of the paper to work the problems!
Collect your work together in numerical order (number each page) by problem when you finish, including your equation sheet at the end, use the envelope provided to store your work and the exam. Good luck!

Please confirm your student ID number: Master.

DO NOT WRITE YOUR NAME!

1. The electric field inside a nonconducting sphere of radius R, containing uniform charge density, is radially directed and has magnitude

\[ E = \frac{qr}{4\pi\varepsilon_0 R^2} \]

where \( q \) is the total charge in the sphere and \( r \) is the distance from the center of the sphere. (a) find the potential \( V \) inside the sphere, taking \( V = 0 \) at \( r = 0 \). (b) What is the difference in electric potential between a point on the surface and the center of the sphere? If \( q \) is positive, which point is at the higher potential? (c) Show that the potential at a distance \( r \) from the center, where \( r < R \), is given by

\[ V = \frac{q(3R^2 - r^2)}{8\pi\varepsilon_0 R^3} \]

where the zero of potential is taken at \( r = \infty \)

2. A resistor is in the shape of a spherical shell, with an inside surface of radius of \( a \) covered with a conducting material and an outside surface of radius \( b \) covered with a conducting material. Assuming a uniform resistivity \( \rho \), calculate the resistance between the conducting surfaces.
3. Show that the plates of a parallel-plate capacitor attract each other with a force given by

\[ F = \frac{q^2}{2\varepsilon_0 A} \]

Prove this by calculating the work necessary to increase the plate separation from \( x \) to \( x + dx \), the charge \( q \) remaining constant.

4. The figure at right shows a “homopolar generator,” a device with a solid conducting disk as rotor. This machine can produce a greater emf than one using wire loop rotors, since it can spin at a much higher angular speed before centrifugal forces disrupt the rotor. (a) Show that the emf produced is given by

\[ E = \pi f BR^2 \]

where \( f \) is the spin frequency, \( R \) the rotor radius, and \( B \) the uniform magnetic field perpendicular to the rotor. (b) Find the torque that must be provided by the motor spinning the rotor when the output current is \( i \).

5. (a) Calculate the magnetic moment of a uniformly charged, rotating sphere. (b) Show that the magnetic moment can be written as \( \mu = qL/2m \) where \( L \) is the angular momentum of the sphere and \( m \) is the mass. (c) Show that this is not a good model for the structure of an electron. (Hint: The uniformly charged sphere must be divided into infinitesimal current loops and an expression for the magnetic moment found by integration.)

6. The AC generator in the figure supplies 170 V (max) at 60 Hz. With the switch open as in the diagram, the resulting current leads the generator emf by 20°. With the switch in position 1 the current lags the generator emf by 10°. When the switch is in position 2 the maximum current is 2.82 A. Find the values of \( R, L, \) and \( C \).
7. A thin, flat plate of partially reflecting glass is a distance \( b \) from a convex mirror. A point source of light \( S \) is placed a distance \( a \) in front of the plate (see the figure) so that its image in the partially reflecting plate coincides with its image in the mirror. If \( b = 7.50 \text{ cm} \) and the focal length of the mirror is \( f = 28.2 \text{ cm} \), find \( a \) and draw the ray diagram.

8. A perfectly flat piece of glass \( (n = 1.50) \) is placed over a perfectly flat piece of black plastic \( (n = 1.20) \) as shown in figure A. They touch at \( A \). Light of wavelength 600 nm is incident normally from above. The location of the dark fringes in the reflected light is shown on the sketch of figure B. (a) How thick is the space between the glass and the plastic at \( B \)? (b) Water \( (n = 1.33) \) seeps into the region between the glass and the plastic. How many dark fringes are seen when all the air has been displaced by water? (The straightness and equal spacing of the fringes is an accurate test of the flatness of the glass.)

9. Consider a planet, with radius \( R \), revolving about the Sun in a circular orbit of radius \( r \). Suppose that the planet has no atmosphere (and therefore no “greenhouse effect” on its surface temperature).

(a) Given that the radiant power output of the sun, \( P_{\text{sun}} \), find an expression for the surface temperature \( T \) of the planet. The Stefan Boltzmann Law states that the intensity of radiation is proportional to \( T^4 \) of radiating body

(b) Evaluate the temperature numerically for the Earth. \((P_{\text{sun}} = 3.86 \times 10^{26} \text{W}, R = 6.37 \times 10^6 \text{m}, r = 1.50 \times 10^{11} \text{m}, \text{radius of the sun } R_s = 6.96 \times 10^8 \text{ m}, \text{temperature of the sun } T_s = 5800 \text{K})\).
10. Show, by analyzing a collision between a photon, and a free electron (using relativistic mechanics), that it is impossible for a photon to give all its energy to the free electron. In other words, the photoelectric effect cannot occur for completely free electrons; the electrons must be bound in a solid or in an atom.