Double Folding Analysis of $^6$Li Elastic and Inelastic Scattering to Low Lying States on $^{208}$Pb

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Abstract

Elastic scattering of 360 MeV $^6$Li ions from $^{208}$Pb was measured from $2.9^\circ \leq \theta_{\text{Lab}} \leq 33^\circ$. The data were fitted with two different potentials, one with the real volume potential folded and one with both real and imaginary volume potentials folded, derived from the M3Y density dependent Nucleon-Nucleon interaction. Distorted Wave Born Approximation (DWBA) calculations were performed with the fitted parameters to calculate cross-sections for inelastic scattering to low lying $3^-$ states.

Introduction

Nuclear incompressibility is the energy needed to change the density of nuclear matter around equilibrium [1]. It is an important parameter of the equation of state (EOS) of nuclear matter and has a variety of applications mainly in the astrophysics field. The nuclear EOS is used to determine the radii of neutron stars [1], predict the dynamics of supernovas, and in simulation of heavy ion collisions [2]. In order to determine the incompressibility of a nucleus, the direct connection between the nuclear incompressibility and the energy of the Isoscalar Giant Monopole Resonance (ISGMR) is exploited [3]. Equation (1) shows the correlation between the energy of this compression mode and the nuclear incompressibility parameter [3]:

$$\bar{E}_{\text{ISGMR}} = \frac{\hbar^2 K_A}{m(r^2)}$$

Where $E_{\text{ISGMR}}$ is the centroid energy of the compression mode, $\hbar$ is the reduced Planck’s constant, $m$ is the nucleon mass, $K_A$ is the nuclear incompressibility, and $(r^2)$ is the ground-
state mean-square radius, with \( \langle r^2 \rangle = \frac{4\pi}{4\pi} \int_0^\infty \rho(r)r^2r^2dr \). Where \( \rho(r) \) is the nuclear matter density distribution with respect to radius.

To extract ISGMR energies, elastic and low-lying inelastic scattering data are used to find and confirm the nuclear potential parameters. These parameters are then used to calculate the differential cross-section of scattering to the various giant resonances. These are then used to perform a multipole decomposition analysis to extract the strength distribution of the ISGMR with respect to excitation energy which then yields the centroid energy of the ISGMR [4].

With Radioactive Ion Beam (RIB) facilities becoming more common we have the opportunity to measure ISGMR energies and strength distributions far from stability. Unfortunately, given the use of inverse kinematics necessitated by RIB facilities one must be careful in choosing the probe [5]. \(^6\)Li is the most appropriate solid target probe because it can be made thin enough for recoils to escape, it has a small mass, and it has an isospin of \( T=0 \) [5]. Unfortunately little is known about using \(^6\)Li as a probe of the ISGMR. As such an experiment to get elastic and inelastic scattering cross-sections was performed with \(^{208}\)Pb using the Grand Raiden spectrometer at RCNP in the University of Osaka, Japan.

Procedure

The raw data was converted into n-tuples using an analyzer written by Dr. Masaru Yosoi of RCNP. Using ROOT [6], the n-tuples were then analyzed. First, a particle ID gate was set on \(^6\)Li particles; next, background regions were defined using the vertical positioning of the particles. Finally, gated and background subtracted spectra of counts vs. scattering angle vs. focal plane x position were obtained. With these spectra the number of counts scattered into each angular
region were obtained giving angular distributions of the differential cross-section for elastic scattering and scattering to the first excited state of $^{208}$Pb.

The unpublished code dfpd4 [7] was then used to compute folded real volume and coulomb potentials for elastic scattering and transition potentials to $^{208}$Pb’s 3$^-$ excited state from the $^6$Li and $^{208}$Pb nuclear density distributions and the M3Y-Paris density [8] dependent nucleon-nucleon interaction. Using the DWBA/coupled channel code ECIS [9], fits were then performed on the normalization of the real volume potential and the depth, radius and diffusivity of the imaginary volume, real surface, and imaginary surface potentials. It is interesting to note that the breakup of $^6$Li is accounted for with the normalization of the folded potential being ~0.65 (as opposed to 1.0). Once a good fit of calculated differential cross-sections to the elastic scattering data had been obtained the parameter set was tested by calculating the angular distribution of the first excited state using the known B(E3) [10] value and the potential parameters. If the calculated distribution was in good agreement with the excited state scattering data then the potential parameters were deemed good.

Additionally, the unpublished code dfpd5 [11] was used to calculate folded real volume, imaginary volume, and coulomb potentials for elastic scattering and transition to the ground state. Once again fits were performed using ECIS, fitting the normalization of the folded imaginary volume potential, and the depth, radius, and diffusivity of the real surface and imaginary surface potentials. At Dr. Khoa’s suggestion, the real volume potential normalization was fixed at 1.0 and the real surface depth was allowed to go negative, producing a repulsive interaction which provided for the break-up for $^6$Li. As before the parameters obtained were testing by using them and the known B(E3) value to calculate excited state distributions for comparison.
Results and Analysis

Hybrid Model 1

This model consisted of a folded real volume potential, a Woods-Saxon imaginary volume potential, and real and imaginary surface Woods-Saxon potentials. The parameters are shown in Table 1.

<table>
<thead>
<tr>
<th>Potential Type</th>
<th>Normalization</th>
<th>Depth (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_0$ (fm)</th>
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<tr>
<td>Real Volume</td>
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<td>–</td>
<td>–</td>
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<tr>
<td>Imaginary Volume</td>
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<td>50.979485</td>
<td>0.814588</td>
<td>1.188500</td>
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<tr>
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<td>0.018280</td>
<td>1.141104</td>
<td>0.861466</td>
</tr>
<tr>
<td>Imaginary Surface</td>
<td>–</td>
<td>10.218913</td>
<td>1.132484</td>
<td>0.763733</td>
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</tbody>
</table>

Table 1: The potential parameters for the first hybrid model.

An initial set of parameters for the real and imaginary volume potentials were found via a brute force search of the parameter space. With those as initial values the full parameter set was fit using more traditional gradient descent $\chi^2$ minimization techniques. The final $\chi^2$ for the elastic scattering was 388. Shown in Figure 1 is a plot of the experimental elastic scattering cross-sections and the calculated differential cross-sections using the Optical Model Parameters (OMP) shown above. Additionally, as a test of their validity the OMP and the transition potentials obtained from them using the B(E3) were used to calculate the differential cross-section for scattering into the $3^-$ state as shown in Figure 2.
Figure 1: The optical model fit for the elastic scattering ($0^+$ state) data with a hybrid double folding fit.

Figure 2: The same optical model with transition potentials for the $3^-$ state taken into account to calculate the inelastic scattering of the data set. As can be seen the calculation does a good job up to $\sim$25 degrees and so the parameters were accepted.
Hybrid Model 2

This model consisted of a folded real volume potential, a folded imaginary volume potential, and real and imaginary surface Woods-Saxon potentials. As stated earlier the real normalization was held at 1.0 and the real surface potential was allowed to become repulsive to provide for the breakup of $^6\text{Li}$. The values obtained for the best fit are given in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Normalization</th>
<th>Depth (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_0$ (fm)</th>
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<td>Real Volume</td>
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<td>Imaginary Volume</td>
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<td>–</td>
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<td>Real Surface</td>
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<tr>
<td>Imaginary Surface</td>
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<td>1.083166</td>
<td>0.295855</td>
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</table>

Table 2: The potential parameters for the second hybrid model.

Figure 3: The optical model fit for the elastic scattering (0+ state) data with the second hybrid model.
As before transition potentials were obtained using the fitted OMP and the known B(E3) value. The results of this calculation are in Figure 4. As can be seen however poor agreement was found between the calculated cross-sections for $3^{-}$ scattering and the experimentally obtained cross-sections showing a need for further refinement of the OMP.

Figure 4: The optical model fit for the $3^{-}$ state scattering data with the second hybrid model.
Conclusion

The optical model parameters found fit the elastic scattering and inelastic scattering data well for the first hybrid double folding model. The optical model parameters for second hybrid double folding model need further refinement as Figure 4 shows.

Future Work

The parameters for the second hybrid model will be improved and once it produces a good match for the elastic and inelastic data, the parameters will be combined with inelastic scattering to find ISGMR energies and strength distributions.

References

[10] D. T. Khoa, computer program DFPD5, private communication