

Structure of even- A nuclei in the neutron-rich region of the
nuclear chart predicted by the r-process.

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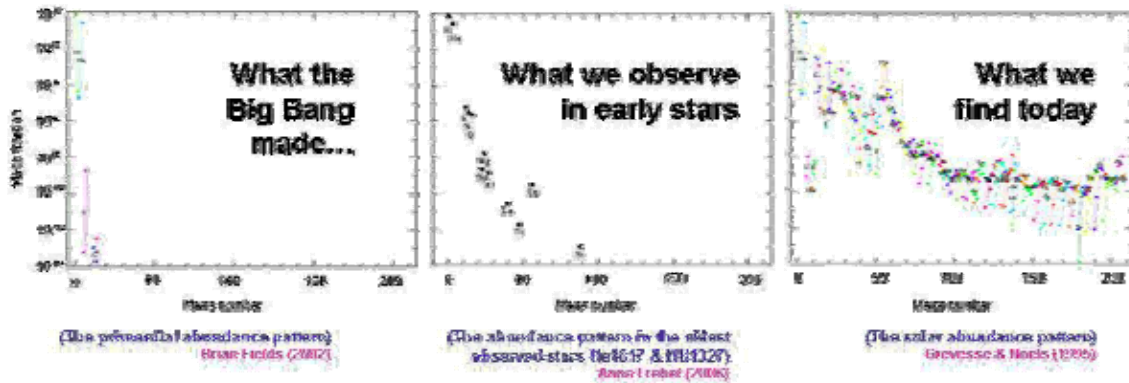
Abstract:

The main goal in nuclear physics is the study of the properties of the nucleus as a function of protons and neutrons that make it up. Three particular areas of interest in nuclear physics are the study of masses, structures and half-lives of nuclei. Much is known about nuclei close to stability but lack of sophisticated equipment has limited research on exotic nuclei in the neutron-rich region of the nuclear chart predicted by the r-process. By studying the energies of the $4+$ and $2+$ excited states of nuclei to the ground state of even- A nuclei, it is possible to roughly determine the structure of nuclei. Looking at how this ratio changes as a function of the P factor, it is possible to see how the structure changes for nuclei as the neutron and proton count move away from close shell magic numbers. This provides an important tool to study the structure of exotic nuclei whose quadrupole moment is very difficult to see experimentally. I will discuss specific predictions on the structure of even nuclei.

Introduction:

Based on our current knowledge, most of the baryonic matter in the universe was formed after the big bang. Matter in the early universe consisted mostly of hydrogen and helium with traces of lithium and beryllium. All other elements up to iron are produced within the cores of stars. This is because very high pressures and temperatures are required for hydrogen to fuse into heavier elements and the cores of stars provide the ideal conditions for the synthesis to happen[1]. However, the Coulomb barrier increases as Z increases and this makes it harder for fusion to synthesize heavier nuclei. Looking at a chart of solar abundances we see the abundances of nuclei $A=1$ to $A=50$ generally drops until it gets close to the iron peak. This is because iron ($A\sim 56$) has greater stability compared to other elements composed from silicon burning. Stable nuclei required much more energy to fuse into heavier elements making this reaction counterproductive for stars whose internal processes are very keen to the energy released from fusion. Fusion alone cannot synthesize all the elements observed. The abundance data first found by Suess and Urey in 1956 and updated since[2] is generally the accepted reference that shows there are other nucleosynthesis processes responsible for element creation. From the data we can observe distinct peaks in abundances for elements in our galaxy with $A\sim 80, 130, 195$. These peaks are representative of magic number of neutrons 50, 82 and 126 and at these values of neutrons it is difficult for nuclei to hold extra neutrons due to their low binding energy. Nuclei heavier than iron are synthesized from two nuclear-astrophysical processes.

fig(1)



To the best of our knowledge the mechanism to create heavier nuclei happens by having iron and other intermediate mass nuclei as seeds with neutron captures and β -decay [3]. These neutrons can collide with seed nuclei and are either absorbed or simply fly straight through. When these neutrons are absorbed, and the new nucleus is stable, it will release the excess energy in the form of gamma radiation and wait for the next neutron to be captured. For each (n, γ) capture reaction we have a nuclei (Z, A) transform into its heavier isotope $(Z, A+1)$ and so on. However, if the resulting isotope is unstable the next reaction is going to depend heavily on the neutron flux interacting with the nuclei and on the lifetime of the nuclei against $\beta(-)$ decay [4]. If the time between neutron captures $\tau_{n\gamma}$ is larger than the β -decay τ_{β} lifetimes the series of networked events are referred to the slow neutron capture or s-process. On the other hand, if the time between successive neutron captures is much smaller than the β -decay, $\tau_{\beta} \gg \tau_{n\gamma}$, then the series of networked events is called the rapid neutron capture or r-process.

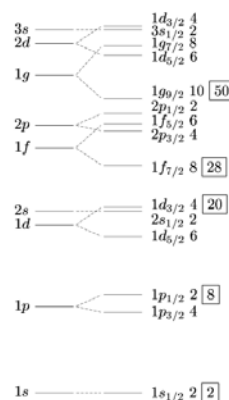
The r-process is believed to be responsible for the creation of about 50% the neutron rich nuclei heavier than iron. Because much is yet unknown about the r-process, there is much interest in the study of masses, structures and half lives. However, the lack of sophisticated equipment makes it very difficult to study exotic nuclei, in part because the small neutron cross sections are very

difficult to achieve and also because the half lives nuclei near the neutron drip line are very short ($t < 3s$). This is why in the recent years there has been a push to construct facilities that can study these nuclei. The FRIB facility at Michigan State University is an example [5].

Shell Model

The work of nuclear astrophysics is to understand the properties of the nucleus as a function of Z and N . An important tool in understanding the properties of nuclei is the shell model (fig.2). The shell model describes the structure of nuclei using energy levels[6]. One of the biggest achievements of the shell model is the predictions of the so called magic numbers (2, 8, 20, 28, 50, 82, 126). These numbers indicate the number of protons and neutrons where shell closures occur. A basic picture of the shell model can be achieved by assuming a single bound nucleon in a square potential with ground solution and a non-zero ground state plus a term proportional to the square of the orbital angular momentum and another term proportional to the orbital angular momentum dotted with the the spin of the nucleon, s .

fig(2):

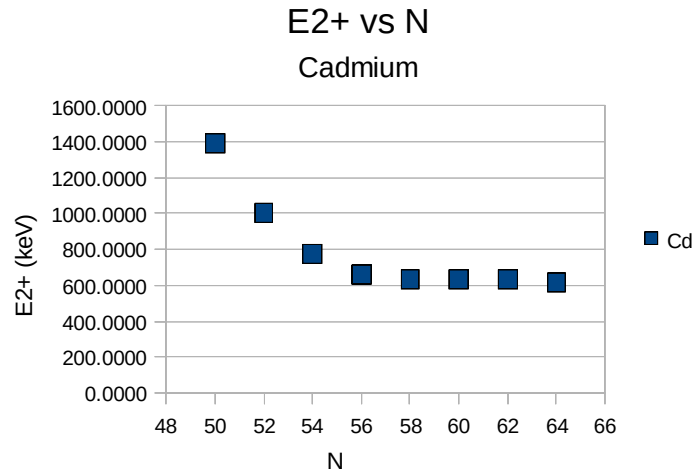


$E = n(\hbar\omega) + l(l+1) + l*s$. The result is the magic number shell closure.

. The energy states come from solving for the eigenvalues in Schrodinger's equation.

Schrodinger's equation:
$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \hat{H} \Psi(\mathbf{r}, t)$$

To excite a nucleon to a higher excited state at the shell closure very high energies are required. This is because at the shell closure the configuration of nuclei is very stable and the binding energy per nucleon is very high. Away from the shell closures this energy decreases. Fig(3):



In the above graph we can see how the excitation energy for the 2+ of Cadmium decreases as the neutron count moves away from the shell closure. This relation can be observed for most nuclei as I will show later. This behavior may change however in the presence of sub-shells, where the energies may be higher and not follow the trend.

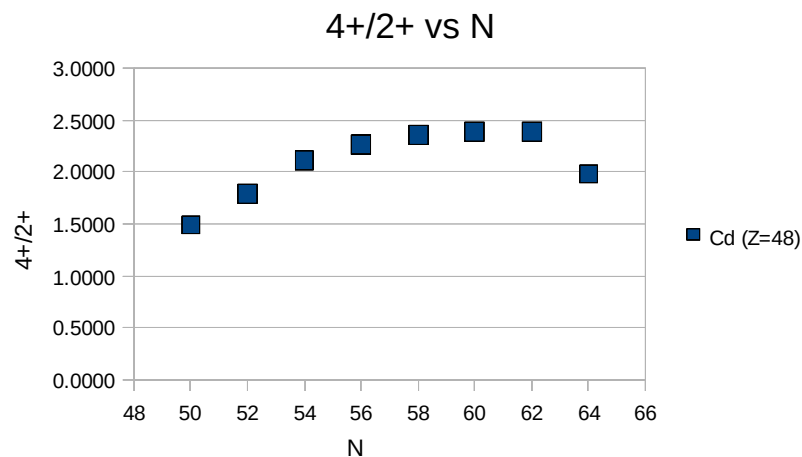
This means that as the number of nucleons increase the number of interactions between protons and neutrons increase. The resulting interactions have a collective effect on the nucleus and eventually start affecting the single particle model. These interactions will allow for multipole terms. The electric quadrupole has a very interesting effect in that an excitation of this term on a

spherical nuclei will cause the nucleus to oscillate in shape[7]. To interpret this phenomena the liquid drop model is used on the nucleus to interpret the oscillations as deformations on the spherical shape of the nucleus [8].

Method

A phonon is the quantized vibrational energy of a multipole ($\hbar\omega$). One phonon quadrupole excitation of the 0^+ state of an even nuclei will produce a 2^+ state and two phonons will produce a 2^+ and a 4^+ state. The energy ratios of the 4^+ and 2^+ states can in this way utilize as an indicator for the collective vibrational effects of nuclei.

Fig(5):



When more p-n interactions occur, higher deformations beyond the vibrational state can happen. At this point the shape of the nucleus can exhibit exotic forms, like prolate and oblate [8]. This means that the nucleus has broken spherical symmetry and can now rotate.

The energies for the ground and low lying states in even nuclei are due to rotation and can be found by the following expression:

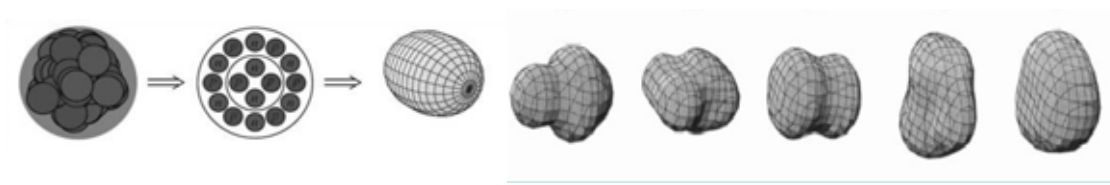
$$E = \frac{\hbar^2 j(j+1)}{2I} \dots \text{where } \hbar \text{ is Planck's constant, } j \text{ is the angular momentum operator for the given excited state and } I \text{ is the}$$

moment of inertia.

For $E_{2+} = 6 \hbar^2 / 2I$

$$E_{4+} = 20 \hbar^2 / 2I.$$

The ratio E_{4+}/E_{2+} is 3.33. This ratio gives us a rough idea about the nature of the nucleus. With ratio less than 2 we have spherical structure. Vibrational state happens when the ratio is 2 and rotational when it is about 3.33.



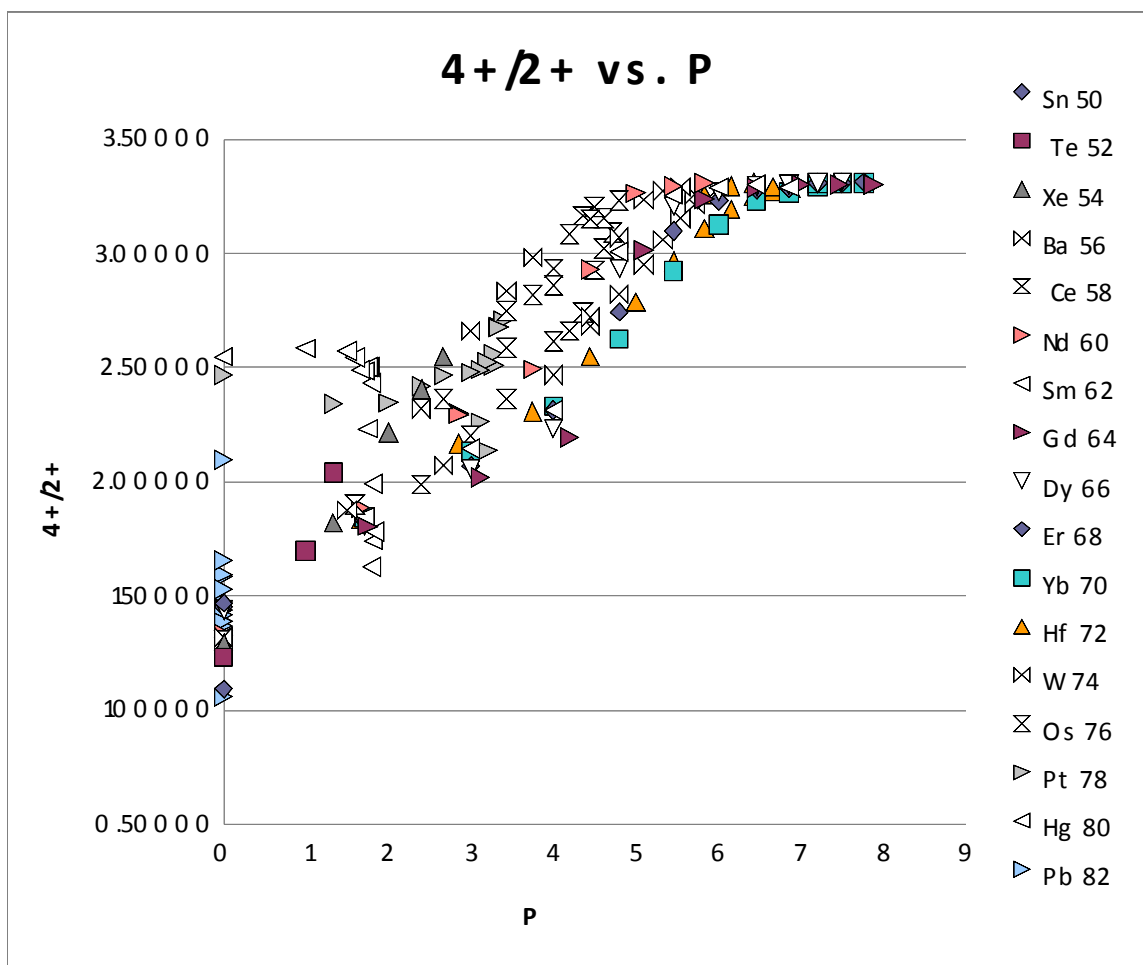
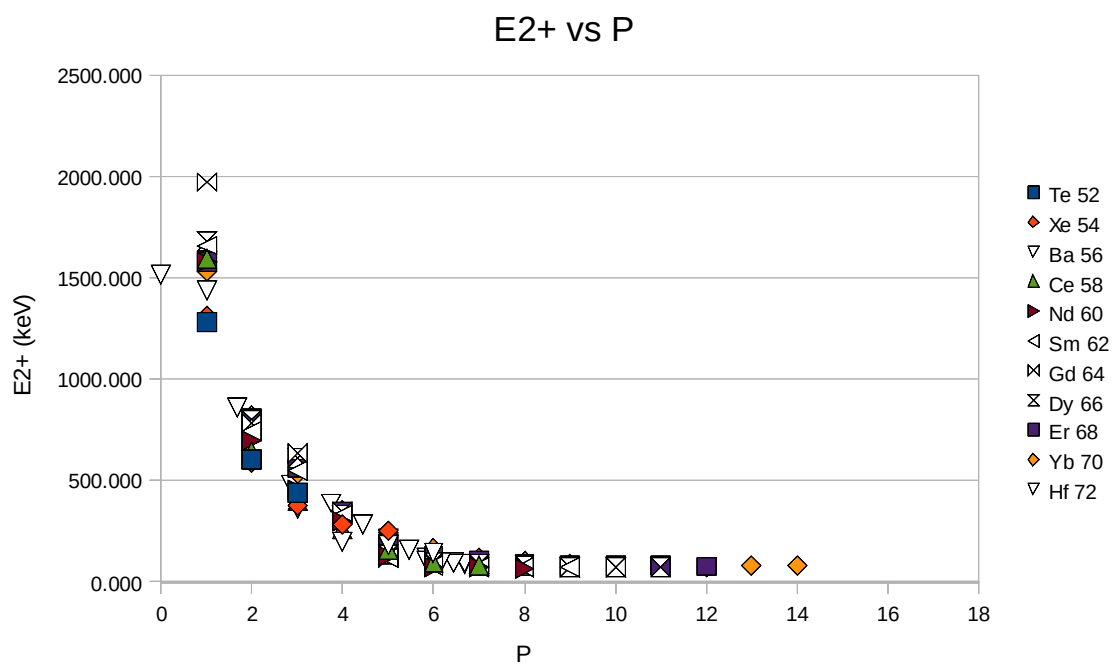
Due to the collective motion of nucleons in a deformed nuclei the E_{2+} will decrease as the number of interactions increases.

A very useful method to study the collective effect of nucleons and deformations is to look at the parametrization of the valence p-n interactions. The $N_p N_n$ value is the product of the number of valence protons and neutrons [9] from the closest shell closure. When plotted against E_{2+} the E_{4+}/E_{2+} and values, it is possible to look at trends in the structure of nuclei. However, when these values (E_{2+} the E_{4+}/E_{2+}) are plotted against the average number of interactions of valence p-n then we get a more general model. This is equal to

$P = (N_p N_n) / (N_p + N_n)$, and is called the 'promiscuity factor' or p-factor for short.

Plotting the E_{2+} and E_{4+}/E_{2+} vs P for nuclei above the $N=82$ shell closure we can observe a trend in the structure:

fig(6)(7):

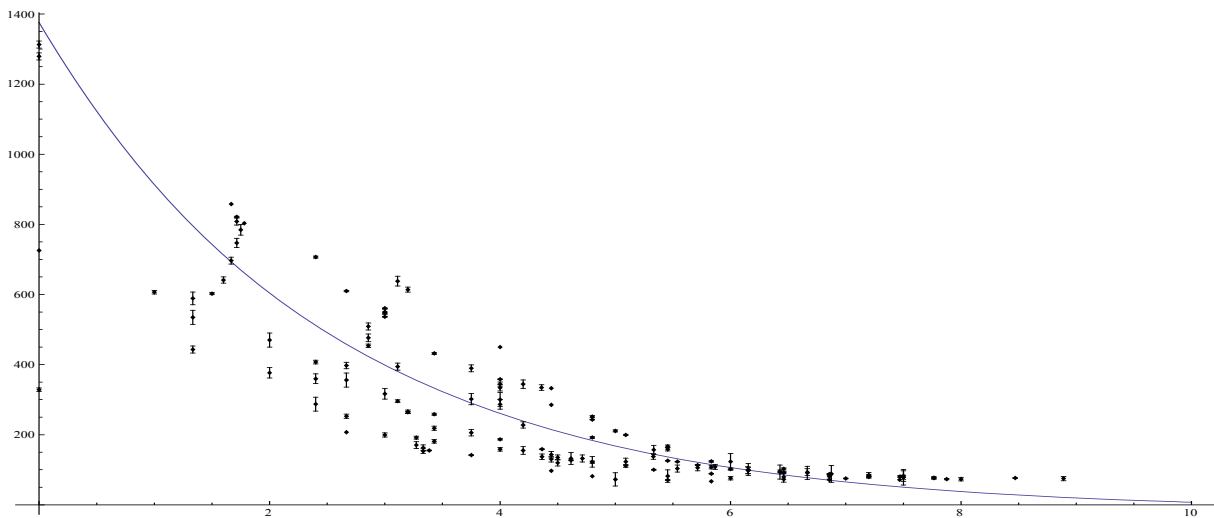


Analysis

From the first plot (fig6) we can see how as P increases the 2+ excitation energies decreases exponentially. Using Mathematica, we can obtain a relation between excitation energy and P:

$$E(2+) = C0 * \text{Exp}[C1 * P] - C2,$$

where $C0=1393.235404483323$, $C1=-0.40331343127832586$, $C2=17.351696566223097$ are constants.



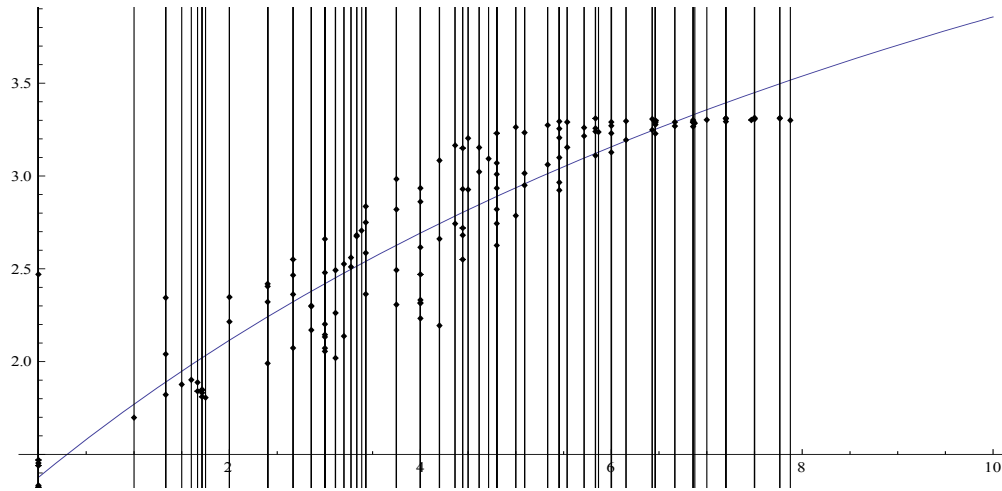
In fig7 we see how the ratio E_{4+}/E_{2+} changes in the form of $1/P$. We can also observe deformations occur when $P > 5$, however there may be deformities when $P=4$.

Once again using Mathematica it is possible to find a function that can interpret the ratio E_{4+}/E_{2+} as a function of P:

$$\text{Ratio} = (-C3/(P+C4))+C5,$$

where $C3=87.21862135032282$, $C4=14.407412308789484$, $C5=7.430532223804219$.

It is important to note that the data obtained for the E_{4+}/E_{2+} ratios have very high errors. It is possible the high errors are due in part because of very short half lives of some of the isotopes makes it very difficult to study for extended periods of time, different measuring techniques are used to detect the low lying energies of nuclei, and the difficulty of creating stable beams of exotic neutron-rich nuclei.



Predictions

With the information that the relation between the ratio(E_{4+}/E_{2+}) vs P provides, it is possible to make predictions about the structures of even nuclei near the N=50 shell closure critical to r-process nucleosynthesis. Cu78, Cu80, Cu79, Ni77, Ni78, Cu77, Zn82 [11] are nuclei near the shell closure that cause bottlenecks in r-process path and are the ones whose theoretical mass models differ the most with observed data. However, the NpNn scheme only allows us to predict structure of even nuclei because the energy level splitting works differently for odd nuclei.

From the table above we see the structure of these nuclei are somewhere between spherical and

	P	E_{4+}/E_{+2}
^{78}Cu	0.99	1.77
^{80}Cu	1.02	1.78
^{78}Ni	0	1.38
^{82}Zn	1.93	2.09

vibrational.

	P	E_{+2}
^{78}Cu	0.99	917.23
^{80}Cu	1.02	905.99
^{78}Ni	0	1375.88
^{82}Zn	1.93	622.34

From the E_{+2} we see the P factor for ^{78}Ni is zero because it is located at the N=50 shell closure and therefore will have a high excitation energy making it difficult to easily absorb another neutron. This contributes to the bottleneck of the r-process path. Out of these four isomers it seems ^{82}Zn is more likely to capture neutrons due its relatively low excitation energy.

Conclusion

The r-process is believed to account for creation of 50% of the neutron-rich atomic nuclei heavier than iron. The main source of observational data for proposed production mechanism comes from abundance distribution of elements in the galaxy. By studying mass abundances, structures and half lives we can attain a better understanding of the network of events that make up the r-process. Studying structure of neutron rich even-A nuclei we can look at where the waiting points in the r-process occur and this helps determine the timescales for the process. The shell model provides a starting point in the study of nuclear structure but as isotopes increase in

As by the capture of neutrons other means to study structure have to be found. The study of low lying excitation energies of even-nuclei produced by the excitation of the quadrupole moments gives us a first clue as to how deformations occur. Plotting E_{4+}/E_{2+} and E_{2+} as a function of the P-factor reveals trends that can show us where deformations happen in the chart of nuclei.

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- [11] Nuclei were identified through a prioritization sensitivity study by Nancy Paul.

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