

QUANTUM STATISTICS IN COMPLEX NETWORKS

Abstract

by

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The Barabási-Albert (BA) model for a complex network shows a characteristic power law connectivity distribution typical of scale free systems. The Ising model on the BA network shows that the ferromagnetic phase transition temperature depends logarithmically on its size. We have introduced a fitness parameter for the BA network which describes the different abilities of nodes to compete for links. This model predicts the formation of a scale free network where each node increases its connectivity in time as a power-law with an exponent depending on its fitness. This model includes the fact that the node connectivity and growth rate do not depend on the node age alone and it reproduces non trivial correlation properties of the Internet. We have proposed a model of bosonic networks by a generalization of the BA model where the properties of quantum statistics can be applied. We have introduced a fitness $\eta_i = e^{-\beta\epsilon_i}$, where the temperature $T = 1/\beta$ is determined by the noise in the system and the energy ϵ_i accounts for qualitative differences of each node for acquiring links. The results of this work show that a power law network with exponent $\gamma = 2$ can give a Bose condensation where a single node grabs a finite fraction of all the links. In order to address the connection with self-organized processes we have introduced a model for a growing Cayley tree that generalizes the dynamics of invasion percolation. At each node we associate a parameter ϵ_i (called energy) such that the probability to grow for each node is given by $\Pi_i \propto e^{\beta\epsilon_i}$, where

$T = 1/3$ is a statistical parameter of the system determined by the noise called the temperature. This model has been solved analytically with a similar mathematical technique as the bosonic scale-free networks and it shows the self organization of the low energy nodes at the interface. In the thermodynamic limit the Fermi distribution describes the probability of the energy distribution at the interface.