FOURTH-ORDER SELF-ENERGY CONTRIBUTION
TO THE TWO LOOP LAMB SHIFT

ABSTRACT
by
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The calculation of the two loop Lamb shift in hydrogenic ions involves the numerical evaluation of ten Feynman diagrams. In this thesis, four fourth-order Feynman diagrams including the pure self-energy contributions are evaluated using exact Dirac-Coulomb propagators, so that higher order binding corrections can be extracted by comparing with the known terms in the $Z\alpha$ expansion. The entire calculation is performed in Feynman gauge.

One of the vacuum polarization diagrams is evaluated in the Uehling approximation. At low $Z$, it is seen to be perturbative in $Z\alpha$, while new predictions for high $Z$ are made.

The calculation of the three self-energy diagrams is reorganized into four terms, which we call the PO, M, F and P terms. The PO term is separately gauge invariant while the latter three form a gauge invariant set.

The PO term is shown to exhibit the most non-perturbative behavior yet encountered in QED at low $Z$, so much so that even at $Z = 1$, the complete result is of the opposite sign as that of the leading term in its $Z\alpha$ expansion. At high $Z$, we agree with an earlier calculation.

The analysis of ultraviolet divergences in the two loop self-energy is complicated by the presence of sub-divergences. All divergences except the self-mass are shown to cancel. The self-mass is then removed by a self-mass counterterm. Parts of the calculation are shown to contain reference state singularities that finally cancel. A numerical regulator to handle these singularities is described.

The M term, an ultraviolet finite quantity, is defined through a subtraction scheme in coordinate space. Being computationally intensive, it is evaluated only at high $Z$, specifically $Z=83$ and 92. The F term involves the evaluation of seven Feynman diagrams with free electron propagators. These are computed for a range of values of $Z$. The P term, also ultraviolet finite, involves Dirac-Coulomb propagators that are best defined in coordinate space, as well as functions associated with the one loop self-energy that are best defined in momentum space. Possible methods of evaluating the P term are discussed.
DEDICATION

This work is dedicated to my parents
Smt. S. Radha and Sri P. M. Ramana Babu.