FOURTH-ORDER SELF-ENERGY CONTRIBUTION TO THE TWO LOOP LAMB SHIFT

ABSTRACT

by

Subrahmanyam Palur Mallampalli

The calculation of the two loop Lamb shift in hydrogenic ions involves the numerical evaluation of ten Feynman diagrams. In this thesis, four fourth-order Feynman diagrams including the pure self-energy contributions are evaluated using exact Dirac-Coulomb propagators, so that higher order binding corrections can be extracted by comparing with the known terms in the $Z\alpha$ expansion. The entire calculation is performed in Feynman gauge.

One of the vacuum polarization diagrams is evaluated in the Uehling approximation. At low Z, it is seen to be perturbative in $Z\alpha$, while new predictions for high Z are made.

The calculation of the three self-energy diagrams is reorganized into four terms, which we call the PO, M, F and P terms. The PO term is separately gauge invariant while the latter three form a gauge invariant set.

The PO term is shown to exhibit the most non-perturbative behavior yet encountered in QED at low Z, so much so that even at $Z = 1$, the complete result is of the opposite sign as that of the leading term in its $Z\alpha$ expansion. At high Z, we agree with an earlier calculation.

The analysis of ultraviolet divergences in the two loop self-energy is complicated by the presence of sub-divergences. All divergences except the self-mass are shown to cancel. The self-mass is then removed by a self-mass counterterm. Parts of the calculation are shown to contain reference state singularities, that finally cancel. A numerical regulator to handle these singularities is described.

The M term, an ultraviolet finite quantity, is defined through a subtraction scheme in coordinate space. Being computationally intensive, it is evaluated only at high Z, specifically Z=83 and 92. The F term involves the evaluation of seven Feynman diagrams with free electron propagators. These are computed for a range of values of Z. The P term, also ultraviolet finite, involves Dirac-Coulomb propagators that are best defined in coordinate space, as well as functions associated with the one loop self-energy that are best defined in momentum space. Possible methods of evaluating the P term are discussed.

DEDICATION

This work is dedicated to my parents
Smt. S. Radha and Sri P. M. Ramana Babu.