

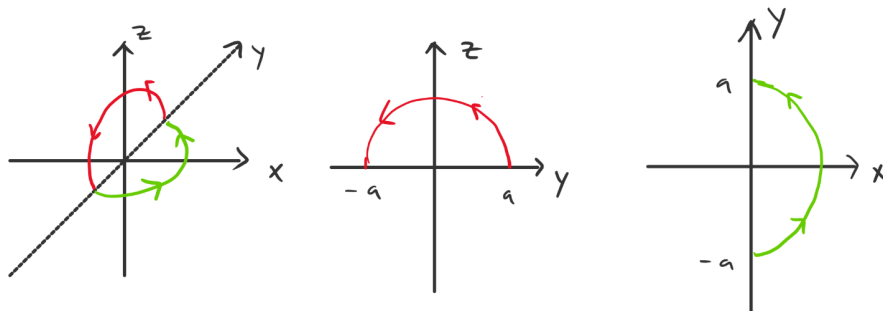
ELECTRICITY & MAGNETISM PRELIMINARY EXAM JANUARY 2025 TEST QUESTION BANK

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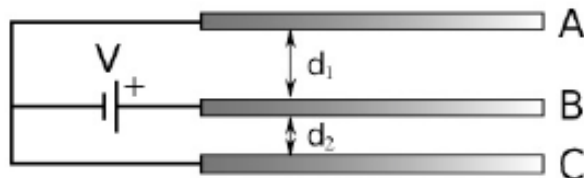
Note: The preliminary examination problems will be drawn from this set. While the spirit and methodology of the problems will be unchanged, some specifics (an initial configuration, what observables are asked for, etc.) may change.

- 1.) Consider a plane linearly polarized monochromatic wave of electric field amplitude E_0 , frequency ω traveling in the direction from the origin to the point $(1, 1, 1)$, with polarization parallel to the xz plane.
 - a.) Find the Cartesian components of the wavevector $\mathbf{k}$ and the unit polarization vector $\mathbf{n}$.
 - b.) Find the electric and magnetic fields as functions of position $\mathbf{r}$ and time t .
 - c.) Find the Poynting vector as a function of r and t .
 - d.) Find the energy density as a function of r and t .

- 2.) You have two semicircular current distributions joined at right angles as shown in the figure. One semicircle is in the $y - z$ plane and the other is in the $x - y$ plane (the first figure shows the full 3-d setup, the second two show the individual 2-d pieces). The radii of the semicircles is $r = a$ and the magnitude of the steady current (moving in the direction of the arrows in the figure) is I . Calculate the magnetic field at the origin $(0,0,0)$. Remember to give both the magnitude and direction.

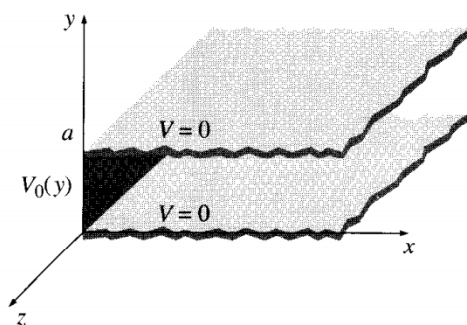


- 3.) Consider three conducting plates each of area A that form two capacitors, as shown in the figure. The outer two plates are at the same potential, the inner one is connected to a battery with potential difference V as shown. Plate A is at a distance d_1 from the Plate B , and Plate B is at a distance d_2 below Plate C .



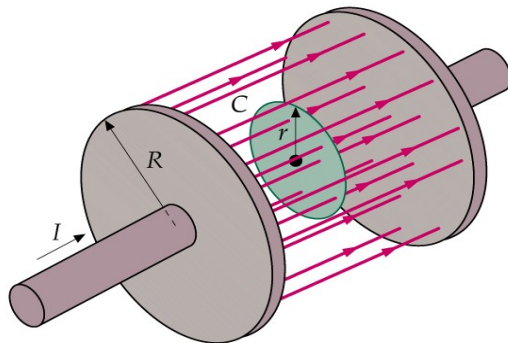
- a.) Are the two capacitors in series or in parallel? Explain your reasoning. Re-draw the figure in your blue book, showing conceptually how the capacitors are arranged in order to justify your argument. (Don't just copy the figure; re-arrange it in a conceptual way so it is clear whether the capacitors are in series or parallel.)
- b.) Suppose the battery has $V = 100$ V, the area of each plate is $A = 0.1$ m². If $d_1 = 9$ mm and $d_2 = 3$ mm, what are the total charges (magnitude and sign) on each of the plates A, B, and C ?

- 4.) You have a pair of grounded conductors arranged as shown in the figure. The space between them is given by a . At $x = 0$, the function $V(y) = V_0 \sin\left(\frac{2\pi}{a}y\right)$ is given. The conducting planes extend out to $\pm\infty$ in z and $+\infty$ for x . We want to find the potential V for the region between the conductors.



- Write down all boundary conditions for this physical scenario.
- Assume a solution that uses separation of variables, $V(x, y) = X(x)Y(y)$. Briefly explain why we can assume the potential has no z dependence.
- Briefly comment on why the solutions for $X(x)$ are exponential functions and $Y(y)$ are sinusoidal functions.
- Use the boundary conditions you wrote in a) to find the final solution for $V(x, y)$. Clearly show your reasoning. Do you need an infinite series of terms or only a finite number of terms to match the boundary conditions?

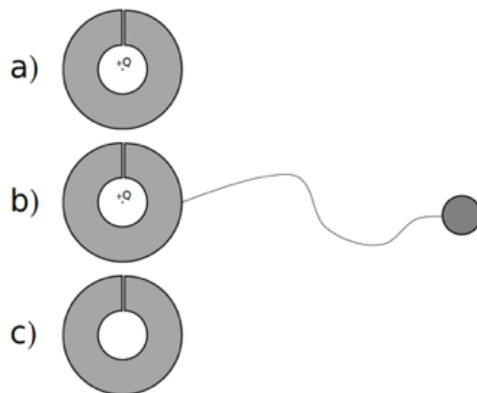
- 5.) A current of I flows into a parallel plate capacitor. The capacitor is formed from two circular plates with radius R , see figure below, and is uncharged at $t = 0$.



- What is the charge on one of the capacitor plates as a function of time?
- Approximating the capacitor as infinitely large, what is the electric field between the plates as a function of time?
- Imagine a circle C with radius $r < R$ placed inside the capacitor and concentric with its axis. What is the electric flux through C as a function of time? Use that flux to find the displacement current flowing through C .
- Apply the generalized form of Ampere's law (also called Maxwell-Ampere law) to the results above to determine the magnetic field (magnitude and direction) at a distance r away from the axis of the capacitor.
- For points r away from the axis of the capacitor, what is the magnitude of the Poynting vector at $t = 1$ s? Does the Poynting vector point towards the central axis or away from it?

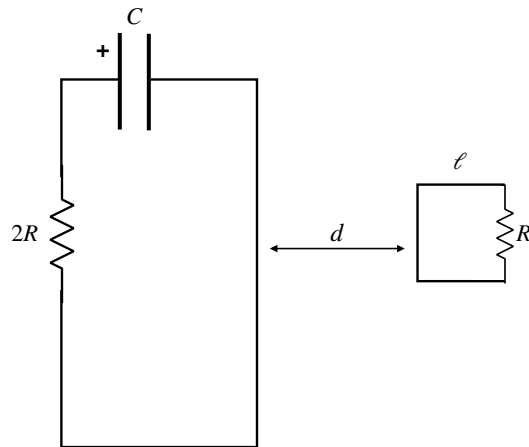
- 6.) A steady current I flows up an infinitely long cylindrical wire of radius R . The current density is given $\vec{J}(r) = k r^2 \hat{z}$ for $r < R$.
- a.) Determine the constant k . What are its (SI) units?
 - b.) Determine what components of the magnetic field are allowed (using cylindrical coordinates (r, ϕ, z)). Briefly explain for each component.
 - c.) Determine if the magnetic field $\vec{B}$ has dependence on the cylindrical coordinates r, ϕ , and/or z . Briefly explain and use this to determine the $\vec{B}$ everywhere for this current distribution.
 - d.) Is there a discontinuity in the magnetic field $\vec{B}$ for this current distribution? If so, explain where it is and what is the magnitude of the discontinuity. If not, explain why not.
 - e.) Show that the vector potential $\vec{A} = \frac{\mu_0}{2\pi} \log\left(\frac{R}{r}\right) \hat{z}$, $r > R$ corresponds to the magnetic field for this current distribution.

- 7.) Suppose there is a thick spherical conducting shell with a small hole bored through to its hollow center. You may assume that the hole is small enough that when analyzing the shell, you may ignore the hole and treat the shell as if it was perfectly spherically symmetric. The shell is initially uncharged.



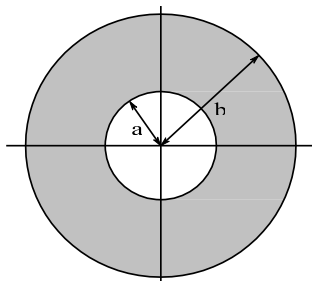
- a.) A point charge $+Q$ is placed at the center of the sphere, as shown in Figure (a). Describe the resulting charge distribution on the shell, both in terms of the locations and amount of any induced charges.
- b.) A long, thin conducting wire is used to connect the shell to a solid conducting sphere whose radius is one-third the outer radius of the shell as in Figure (b). Now, describe the charge distributed on both the shell and the sphere both in terms of location and amount. You may assume that the spheres connected by the wire are very far apart.
- c.) Finally, the wire is removed, detaching the sphere from the shell. After this, the point charge is removed from the center of the shell. In terms of location and amount, describe any remaining charge distribution on the conducting shell.

- 8.) Two loops are placed next to each other. The first loop contains an RC circuit with capacitance C , resistance $2R$ and width L . The capacitor is initially fully charged (charge Q_0) and begins to discharge at $t = 0$. The second loop is located a distance d away from the RC loop. It is a square loop with sides of length ℓ and has a total resistance R .



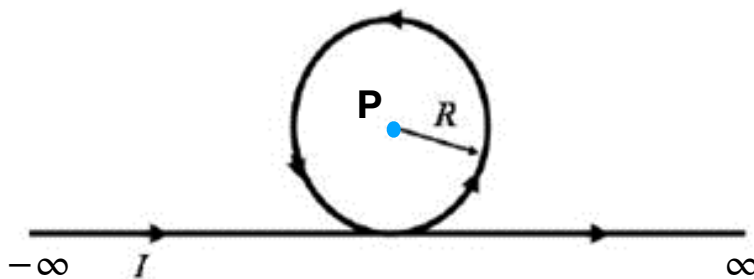
- What is the current in the RC loop as a function of time?
- Treating the sides of the RC loop as infinitely long, calculate the magnetic flux through the second loop as a function of time.
- Use the flux in part b.) to determine the induced current in the square loop. In which direction does the current flow?

- 9.) A total amount of positive charge $+Q$ is spread onto a non-conducting, flat, circular annulus (e.g. “donut”) of inner radius a and outer radius b . The charge is distributed so that the charge density (charge per unit area) is given by $\sigma = C/r^3$, where r is the distance from the center of the annulus to any point on it.



- a.) Find the value of C in terms of Q, a, b and numerical constants. [You may not need to use all these variables.]
- b.) Find the electric potential at the center of the annulus. Take $V = 0$ at $r \rightarrow \infty$. The constant C should not appear in your answer, but the charge Q , the distances a and b , as well as numerical constants may.

- 10.) An infinitely long, straight wire is bent as shown in the figure below and carries a current I . The circular portion has a radius of R . Find the total magnetic field at the center of the loop – point P – using the steps given below:



- Obtain the magnitude of the magnetic field at point P due to the loop of wire. Write your answer in terms of the current I , the radius R and the constant μ_0 . Does this field point into or out of the page?
- Obtain the magnitude of the magnetic field at point P due to the straight part of the wire. Write your answer in terms of the current I , the radius R and the constant μ_0 . Does this field point into or out of the page?
- Combine the results of the previous parts to determine the total magnetic field at point P .

11.) A solid dielectric sphere, with a radius R , has a uniform frozen-in polarization of $P = P_0 \hat{z}$, where P_0 is a constant.

a.) Calculate all bound charges for the sphere. Make a sketch of the bound charge.

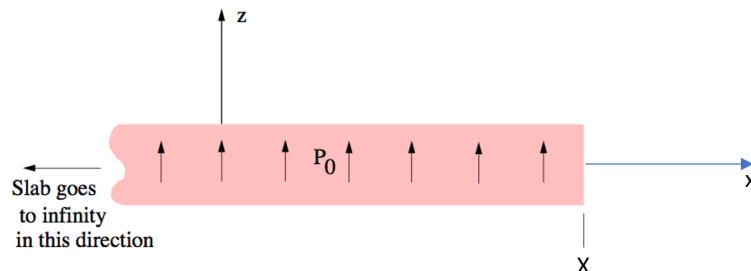
b.) The potential V for this sphere is

$$\begin{cases} V = \frac{P_0}{3\epsilon_0} r \cos \theta; r \leq R \\ V = \frac{P_0}{3\epsilon_0} \frac{R^3}{r^2} \cos \theta; r \geq R \end{cases} \quad (1)$$

What is the electric field E inside the sphere?

c.) Now imagine that you have an “inverse” situation where you have a uniform polarization everywhere given by $P = P_0 \hat{z}$, and you have a hollow spherical cavity of radius R hollowed out. This time, assume the polarization is induced by a polarizing field E_0 and the displacement field is given by $D_0 = \epsilon_0 E_0 + P$. Find the electric field E in the center of the cavity. Also find the displacement field D in terms of E_0 and D_0 .

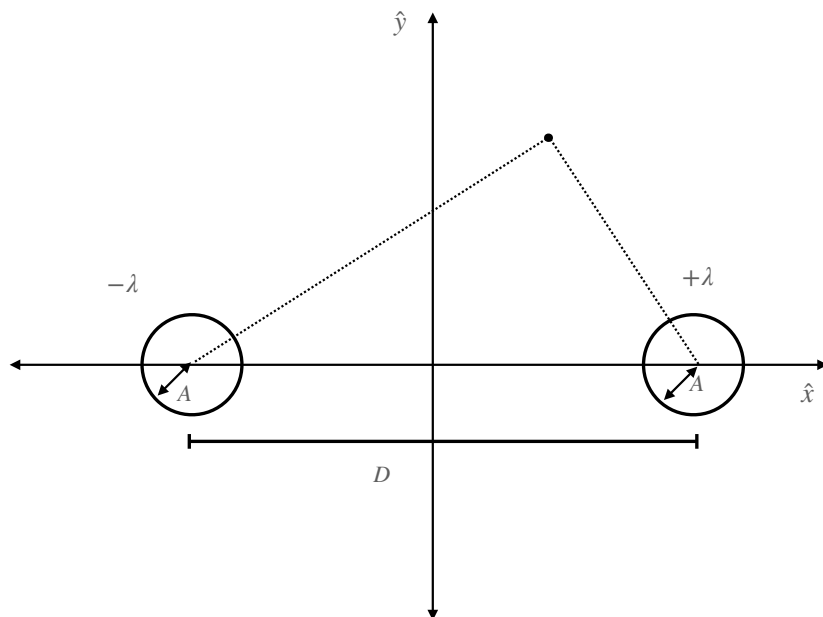
- 12.) A large slab of a dielectric with a permanent “frozen-in” polarization $P = P_0 \hat{z}$. It extends to $-\infty$ in but extends to X in the positive x direction.



For each of the following, also briefly describe the reasoning for your answer.

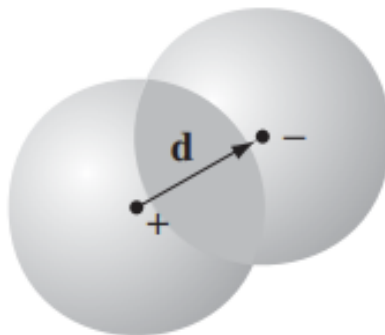
- Make a sketch of the slab and indicate any bound or free charge.
- On the same sketch, draw vectors that represent the electric field E and the displacement field D . The relative length of the vectors should show the relative strength of the fields.
- Indicate regions where you expect the E field or D field to be zero.
- Indicate regions where you expect $\nabla \times E$ or $\nabla \times D$ to be zero.

- 13.) You have two infinitely long metal cylinders of radius A separated by a distance D . One of the cylinders have a charge per unit length λ and the other has a charge per unit length of $-\lambda$. Assume $A \ll D$ so that the charge distribution on each cylinder is unchanged by the presence of the other cylinder.



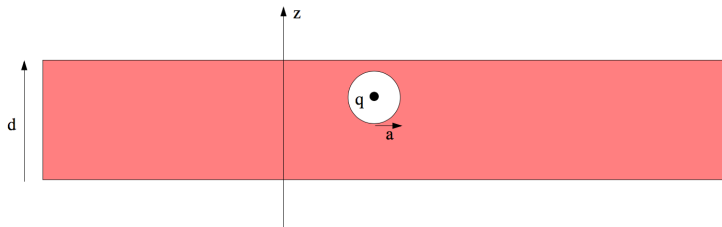
- Find the electric field of this charge configuration everywhere in space.
[Hint: use superposition!]
- Write the potential everywhere outside the cylinders, taking the potential to be zero at the origin.
- What is the capacitance per unit length of this configuration?

- 14.) Two spheres, each of radius R and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap as show in the figure below.



The vector from the positive center to the negative center is d . Show that the field in the overlap region is constant and find its value.

- 15.) A large (approximately infinite area) conducting slab of thickness d and area A where $A \gg d$, has a spherical cavity of radius a . Inside this cavity is a charge $+q$ at its center. The slab has a net charge $+Q$.



- Find the surface charge on the inner and outer surfaces of the cavity. Ignore the effects of the edge of the slab. What is the total charge on the outer surface of the slab?
- Find the electric field i.) outside the slab, ii.) inside the bulk of the slab, iii.) inside the cavity. Make a sketch of the field for all regions, and clearly show your reasoning and calculations.
- Now, we ground the slab (slab is at $V = 0$) and the outer charge is drained away ($q = 0$ on the outer part of the slab). Is there a change in the: i.) inner cavity charge density?, ii.) electric field outside the slab?, iii.) electric field inside the cavity? Explain your reasoning for each of your answers.

- 16.) You are given a spherical shell with a radius R that has a given surface charge distribution given by $\sigma(\theta)$. The potential $V(r)$ is given by

$$V_{\text{out}}(r) = V_0 R^2 \frac{\cos \theta}{r^2},$$

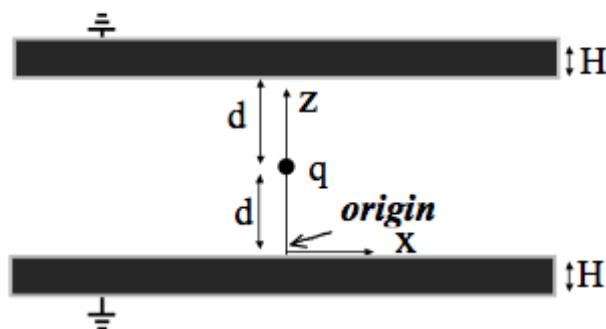
outside the shell and

$$V_{\text{in}}(r) = C r \frac{\cos \theta}{R}$$

inside the shell.

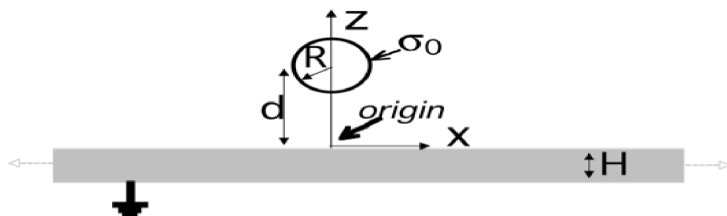
- a.) Determine the constant C in the expression for the potential inside the shell.
- b.) Find the surface charge density $\sigma(\theta)$.
- c.) What is the net charge on the sphere? Briefly show your reasoning.
- d.) What is the dipole moment $\vec{p}$ of the sphere? Explain your reasoning and give the magnitude and direction of $\vec{p}$.

- 17.) You have infinitely wide grounded conducting plates that have a finite thickness as shown in the figure.



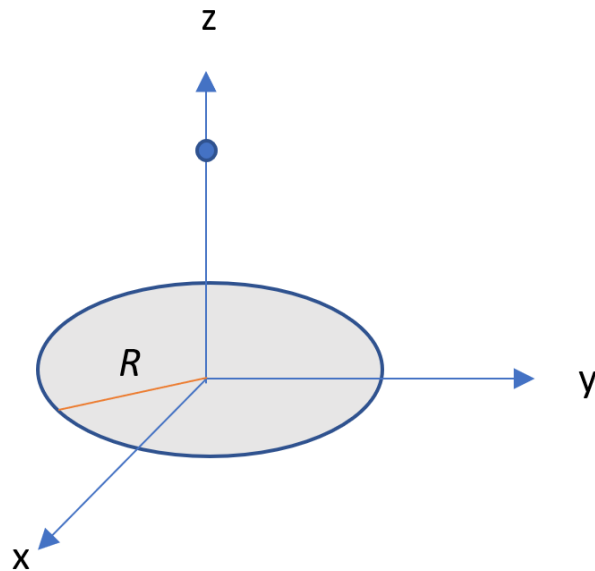
We want to find the voltage $V(r)$ in the space between the conductors. A friend proposes to use the method of images. Their idea is to put i) a negative charge $-q$ a distance d above the bottom surface of the top plate (at $z = 3d$) and ii) put a negative charge $-q$ at a distance d below the top surface of the bottom plate (at $z = -d$). Is the potential of the image charge distribution proposed a valid solution for the original problem? Briefly justify your answer.

- 18.) You have a small thin plastic shell with radius R and a uniform surface charge density σ_0 . The center of this sphere is fixed at a distance " d " above the center of an infinite grounded conducting plate. The conducting plate has a thickness H .



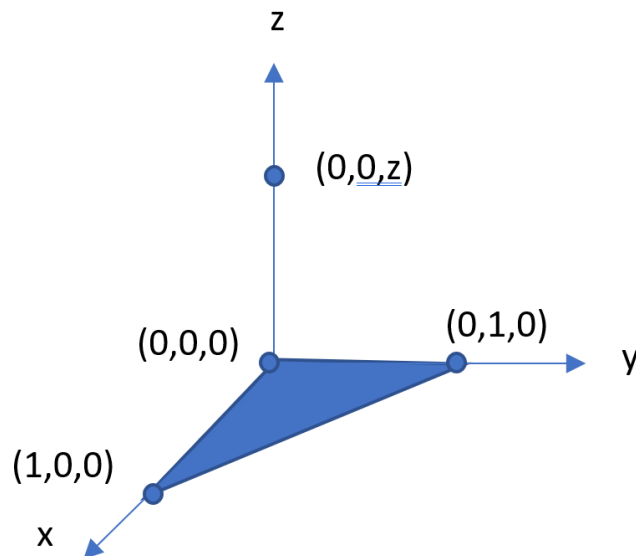
- a.) Find the voltage $V(x, y, z)$ at all points in space, both above, in, and below the plate. Explain and show your work. *Note: that there is one rather tricky region you may need to think about carefully, and describe separately: namely, V inside the sphere.*
- b.) Find the surface charge density $\sigma(x, y)$ on the surfaces of the conducting plate in the previous question. (What is it on the top surface and on the bottom surface of the plate?)

19.) You have a disk on the xy plane with a radius R , as shown below.



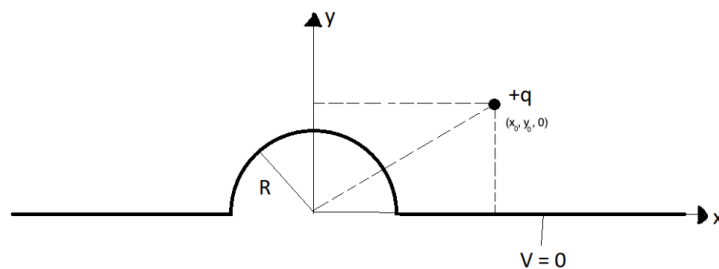
The disk has a surface charge density given by $\sigma(s) = \frac{\alpha}{s}$. Find the electric field at a point on the z -axis above the charge distribution. Include both the magnitude and direction.

- 20.) You have a uniform surface charge density σ on the xy -plane with a triangular shape. The charge density is bounded by the points $(0, 0, 0)$, $(1, 0, 0)$, and $(0, 1, 0)$.



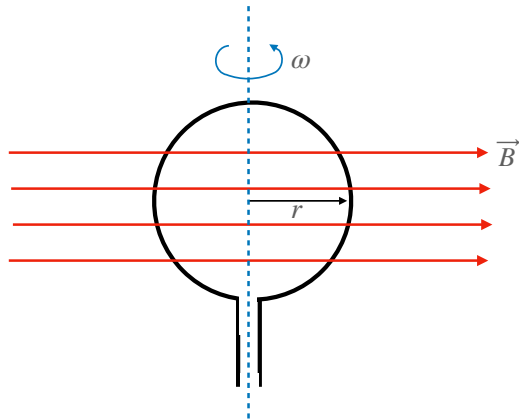
Write an integral expression for the potential V at the point $(0, 0, z)$. You do not have to evaluate the integral, but it must be in a form that can be evaluated using integral tables or a computer algebra program such as Mathematica. Clearly show all steps in getting your final answer.

- 21.) You have a semi-spherical conductor imbedded in an infinitely wide conducting sheet. The point charge q is placed in the position shown above at $(x_0, y_0, 0)$.



You can find the potential V everywhere outside the conductors above the xz -plane using the method of images. Sketch the positions of the image charges. Based on the position of the original charge, give the quantitative positions and charges of all the image charges.

22.) A wire loop is rotated at constant angular velocity in a uniform magnetic field.



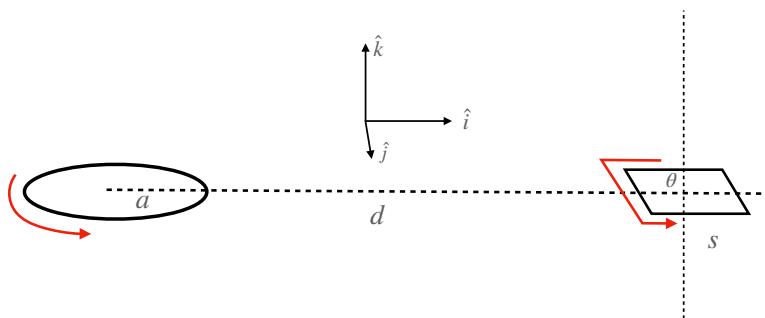
Write an equation for the time-dependent voltage in the wire.

23.) The Biot- Savart law in terms of vector potential is:

$$\vec{A} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l}}{|\vec{r} - \vec{r}'|}$$

Derive the magnetic field $\vec{B}$.

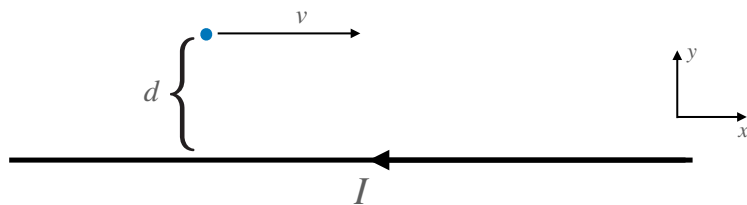
- 24.) Consider two current loops: one circular and one square, separated by a distance d .



The circular loop is held in place. The square loop has mass m . The moment of inertia of the square loop about the midplane is $\frac{ms^2}{12}$. Answer in terms of l, a, d, m , and θ . You may take $d \gg a, d \gg s$.

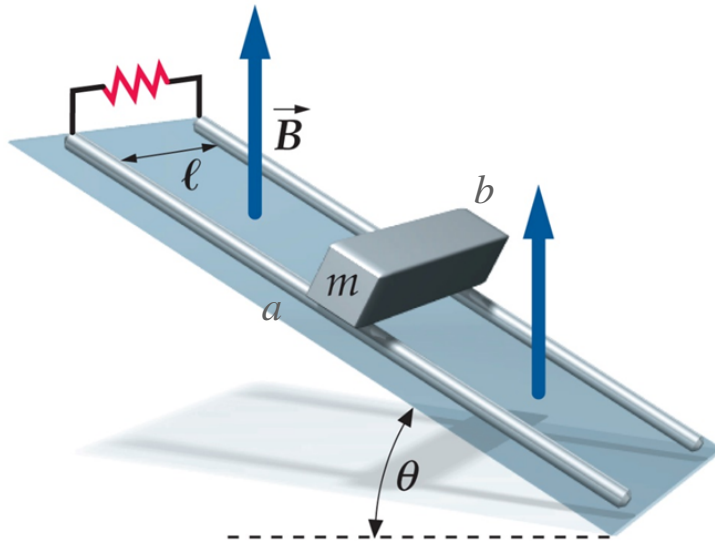
- Each loop is given a current l . What are the magnitude and direction of the magnetic field induced by the circular loop at the location of the square loop?
- The square loop is originally in the same plane as the circular loop ($\theta = 0$). It is then nudged to have a small nonzero θ . What is the torque on the square loop?
- Write (but do not solve) a differential equation for θ .

- 25.) An electron moves parallel to a wire carrying current I . The electron's distance and velocity are initially d and v .



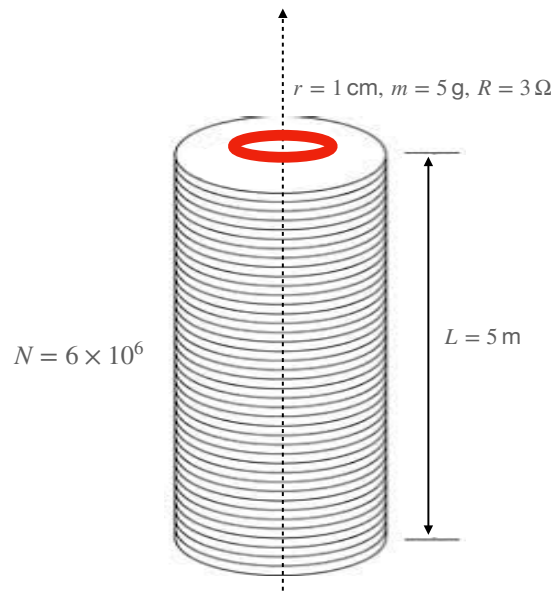
Write (but do not solve) equations describing the electron's trajectory in x and y . There should be two coupled ordinary differential equations.

- 26.) A metal bar with length ℓ , mass m and resistance R is placed on frictionless rails that are inclined at an angle θ above the horizontal. The rails have negligible resistance. A uniform magnetic field of magnitude B is directed upwards, as shown in the figure. The bar is released from rest and slides down the rails.



- a.) What direction is the current induced in the bar – from points labeled a to b or from b to a ?
- b.) What is the terminal speed of the bar?

- 27.) A small metal ring is placed at the center of a long solenoid with 600,000 turns of wire.

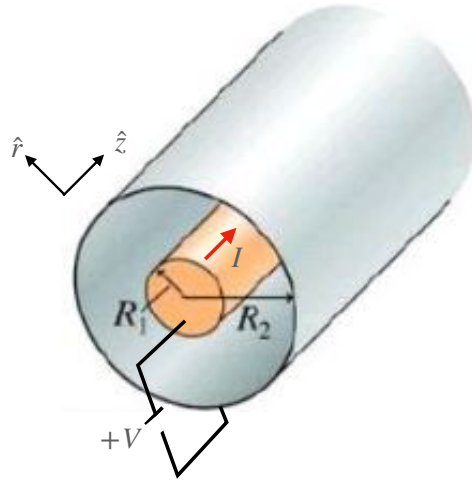


The ring has a radius of 1 cm, a mass of 5 g, and a resistance of 5Ω . The solenoid has a length of 5 m. A current of 9 kA is suddenly applied to the solenoid, then turned off after 3 ms.

- What is the current induced in the ring?
- What force is applied to the ring during the time that the current is on?
- How high does the ring jump?

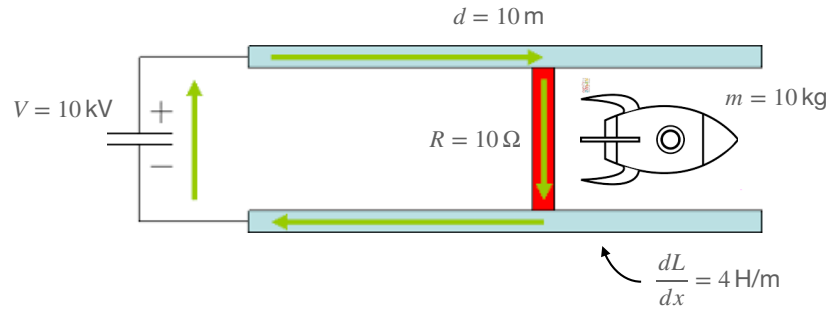
- 28.) Derive the electromagnetic wave equation for E and B from Maxwell's equations in free space.

- 29.) Consider a coaxial cable. It has an inner wire of radius R_1 with a current I . A non-conductive medium separates the inner wire from a conductive sheath. The sheath has an inner radius R_2 . The wire and sheath are held at a potential difference of $+V$.



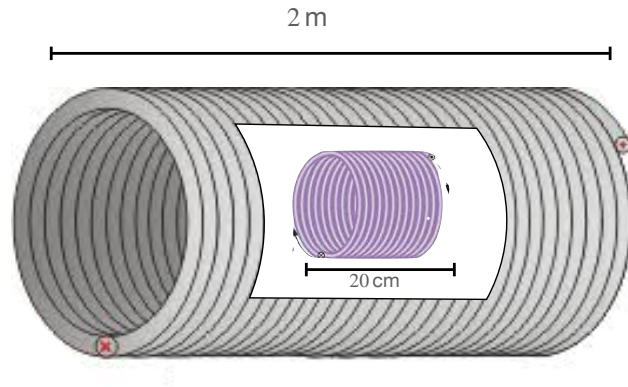
- Find the electric field between the wire and the sheath as a function of r .
- Find the magnetic field as a function of r .
- Use Poynting's theorem to find the enclosed power transmitted as a function of r .
- What is the total power enclosed at R_2 ?

- 30.) Consider a rail gun. A small rocket (10 kg) is launched by a rail gun of length 10 m. The rails have inductance per length of 4H/m. The circuit has 10kV through a resistance of 10Ω .



- a.) What is the force on the rocket?
- b.) What is the exit velocity of the rocket?

- 31.) A 2 m long solenoid with 300 turns of wire has a resistance of 50Ω . It encloses a different solenoid that is 20 cm long, 10 cm wide with 100 turns of wire and a resistance of 20Ω .



An AC voltage (120 V, 60 Hz) is applied to the inner solenoid. What is the current as a function of time in the outer solenoid? *Hint: use the principle of mutual inductance.*